AN ADVANCE STUDY OF NON-UNIFORM RATIONAL BASIS SPLINE CURVES & SURFACES

"SUPERQUADRICS"

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CERTIFICATE

It is certified that the work contained in this thesis entitled **“An Advance Study Of Non-Uniform Rational Basis Spline Curves & Surfaces – Superquadrics”,** by **Hem Raj Yadav** (Roll No.1180456001), for the award of **Master of Technology** from Babu Banarasi Das University has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

Today various methods exist for object representation. And many new methods are being introduced day by day for various branches of engineering. Object representation means the presentation of any object with a maximum accuracy of size and shape. Whether, the object is 2-D or 3-D, real or imaginary.

Among the basic problems recorded in pre-state of this research is the various complication in complex design & analysis, such as, need to learn about lots of tools, settings, and multiple software; lengthy time requirement for engineering design due to repetitive work; and, expensive and time-consuming design and analysis procedures. Additionally, quality aspects also put in this section.

To optimize the quality aspects, competition can be seen among such various methods of object representation. And as a result, there are many computer graphics software for various object representations. The capability and quality aspects of any method of object representation depend on the basic phenomenon. Where, the Non-Uniform Rational Basis Spline Curves and Surfaces (NURBS) is a large group of curves and surfaces, which able to represent any 2-D structure as well as surfaces; There, Superquadrics is the large, joint and hybrid family of various shapes and solids, which able to represent any 3-D object easily using primitive phenomenon. That's the interesting reason to choose them in my study, also as I hope they can change the game of design in various filed of engineering and art; And I feel free to say it based on this study and its findings.

Thus, the proposed work aims to do a deep study on the Non-Uniform Rational Basis Spline Curves and Surfaces (NURBS) and especially on Superquadrics. This research represents several applications for the design and modeling of various objects used in computer vision, computer graphics, reconstruction, robotics, and aerospace engineering.

The computer-aided graphics is well known to design, analyze, simulate, and optimize the different parameters of the quality aspect of the design of complex structured parts & products. Therefore, the main goal is set to find an optimized result, to enhance the quality of designs and models of the mechanical and aerospace industry using it.

The thesis focuses on the superquadrics representation of 3-D complex objects and multi-object scenes. Three issues motivate this research. First, the representation of a complex objects or multi-object scenes has not been easy due to their complex geometry. Second, there is a large scope of the world of superquadric primitives in computer graphics, but still, limited applications are being found. and third is that still today after using computer graphics the design and modeling of a complex

object is a tough task and it not only for beginners but also hard for experienced engineers, because software operations are not properly set up in their minds due to tools technical complexity.

In summary, the four main findings are introduced under this thesis, in which one is that the mathematical modeling is the better method of modeling; second, achieved variable shapes due to the effect of a variable quantity of grids and polylines called as Gridded Superquadrics (GSq); third, an introduction of new, third exponent in the mathematical equations of superquadrics named as Tri Exponential Superquadrics or Superquadrics with Third Exponent (SqWTE), Thus the new family is also categorized into Even Superquadrics (ESq) and Odd Superquadrics (OSq); and fourth is the range of valid and invalid value of these exponents of superquadrics equations.

Also, the theoretical and experimental results as well as mathematical analyses are verified with previously available research. Where, the literature study includes the study of around hundred of research papers, several books, patents, project reports, thesis, and web media also, for core and related topics of the subject.

Finally, the thesis concludes a summary and suggests using the superquadrics model to save much time and cost in the designing and modeling of any complex geometric structure, parts, and products.

Mathematical representation of tightening aspect of surface joint in geometrical modeling by using tightness factor to clarify the limit error; Geometrical modeling of remaining gridded superquadrics i.e. superhyperboloids and supertoroids; and to summarize all these achieved gridded superquadrics, tri exponential superquadrics; And make them useful by presenting them in some examples like; There are, some tasks are being set for future studies and research in the closing of the thesis.

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I have never seen The God, but somewhere, he is being seen help me, ever since I got my own mind and understanding, and before it too; They are in my soul, body and with me too, as on my tongue; and they help me on various ways as per the instantaneous requirements in every difficult time and every situation; And if we call them, The Father of The Gods (Param Pita Parmeshwar), then all these persons, appear as integral parts of that God; I, my lives, life, my values, honours, pride and my any activity or greatness are not possible in the absence of their various help. Indeed, from the depth of heart, with full of emotions, a great thank you to all of you.

(Hem Raj Yadav)

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LIST OF SYMBOLS, ABBREVIATIONS AND NOMENCLATURE

Short Form

BOT : Base and Organization of Thesis	vi, 6
B-Rep : Boundary Representation	9
CAD : Computer Aided Design	1, 2, 11, 15
CSG : Constructive Solid Geometry	9, 64
E-DMAA : Engineering Design, Modeling and Analysis	5
ESq : Even Superquadrics	iv, 49, 54, 56
ESt : Even Supertoroids	55
GI : General Introduction	1
GSq : Gridded Superquadrics	iv, 43
GUI : Graphical User Interface	15
MPSq : Mathematical Preliminaries of Superquadrics	31
NURBS : Non Uniform Rational Basis Spline Curves and Surface	iii, iv, 12, 14
OSq : Odd Superquadrics	iv, 49, 55, 56
OSt : Odd Supertoroids	55
PMSq : Polymodels of Superquadrics	43, 56
Se-WGS : Superellipsoids with Grid size	45
Sh1-WTE : Superhyperboloids of 1 sheet with Third Exponent	51
Sh2-WTE : Superhyperboloids of 2 sheet with Third Exponent	52
SH-WTE : Superhyperboloids with Third Exponents	51
SqWTE : Superquadrics with Third Exponent	-
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CHAPTER 1: INTRODUCTION

1.1 General Introduction (GI)

Computer, leave today, seeing its past - present, it seems that tomorrow it will be the only kingpin and in the future, it will be the only god. Yes, indeed, we can now easily realize that all people consider it not like a machine but as a dream deity and also, not only in the imaginary or virtual world but also in the physical and real-world, It too, Proving impossible things. And, the changing society of living, working and studying can be seen anywhere due to this.

For Design Engineers, It makes it easier from easy, to draft, design, model, analyze, represent, manufacture, and other various tasks, just like as for other professionals and fields.

Today, Computer Graphics is the most important part of all kinds of engineering, correlated either directly or indirectly. Actually, it has become a part of our life and not only of business life but also of whole life. Day to day, from waking up to sleeping, and then till waking up, every moment we need it now. Through various ways, such as mobile phone, personal computer, office computer, car display, aircraft avionics, convergence and vision in robotics, space shuttle simulation, satellite image processing, electronic control units, various computer industry, and many other places, we can see its applications and requirements. After its invention year i.e. 1960, by Verne Hudson and William Fetter, It has raised its flag in each and every field.

As we all know, computer-aided design (CAD) is the most important for all design engineers, and without it, we never find ourselves as a design engineer, No matter how we are, whatever, but, nonetheless, it seems a bit incompleteness, when we claim us as a design engineer and do not have the knowledge of Computer-aided Design. That is also a basic reason to choose the field of study.

3-D design and Modeling, Surface and Solid Rendering, Image Processing, Computational Geometry, Vision and photography, Animation, Video gaming, robotics, automation, simulation, animated movie design, stimulation, advertising, and many others applications are of the computer graphics, Out of that, here our topic is 3-D Geometrical Design and Modeling (GDM). Such a design or model depends on various mathematical representations, mathematical analysis, and various kinds of topology. For example wireframe modeling, surface modeling, and solid modeling, all have a different mathematical, geometrical, and topological view. As it is clear by name '3-D Geometrical Design and Modeling (GDM), it deals with mathematics, computational geometry, topological analysis, and computer graphics for designs and models of various objects. If it is about design, modeling, and its mathematics, then curve and surface are the two main points of study. And in this line, if we come forward for advancement, we come to the Non-Uniform Rational Basis Spline Curves and at depth, we find the Superquadrics. That is the focus of this thesis.

1.2 Modern Research Issues (MRI)

Despite the extraordinary improvement in designing and modeling procedures, in the form of its digitalization and many up-gradation of various CAD software, we have various issues in this field, such as

1.2.1 Various Complexities of Design & Analysis

Various complications, that comes during designing and modeling of complex objects due to their complex morphology and other specific requirements.

Fig. 1.1 Presentation of the result of various problems and complexities

For example, complications related to curve and control points, that disable us to make the required micro changes in design, or reduce the control on the parts of curve and surface, and other aspects related to surface quality due to rendering, and many other giants and comprehensive issues can be seen during such a complex design or modeling.

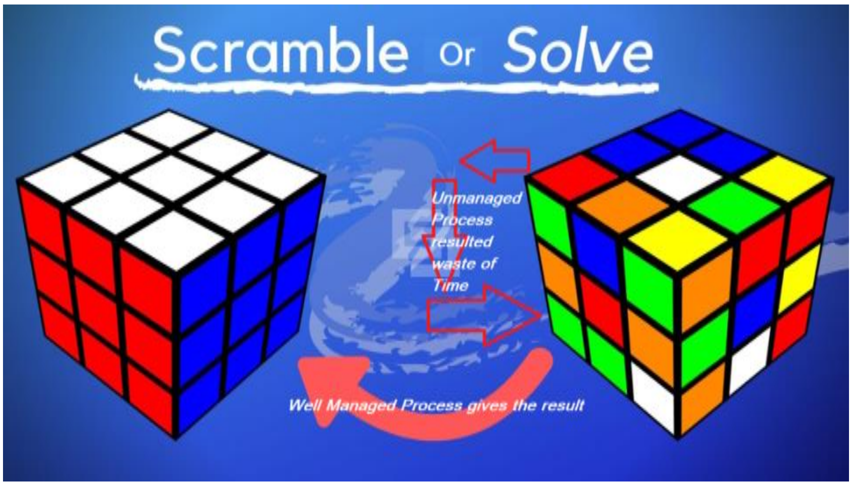

Fig. 1.2 Presentation of Time in respect of process, Unmanaged Process resulted in Waste of Time (WOT), and a Well Managed Process (WMP) produces the Required Result within a minimum period of Time

1.2.1 Heavy Time Limit of Engineering Design Procedure

Due to the long and lengthy procedure of design and modeling, we encounter time problem that means for the design of a small part we need some time, while before it that part had been designed many times by many other people but due to disarrangement we getting to do that design and like us many other people going to for same and in such a way there is a heavy loss of time and cost also, But if we remove it and put that design in a specified data then, it will be so easy, that, we can use that design frequently without any struggling or doing any work again and again. This concept is promoted here in this thesis.

1.2.2 Repetitive Design Work

As we have discussed just in the above section, due to the repetition of work we are losing manpower, energy, time, and a heavy cost. Therefore, it is the giant problem of today's global environment of design and modeling world or of the global system of design and modeling.

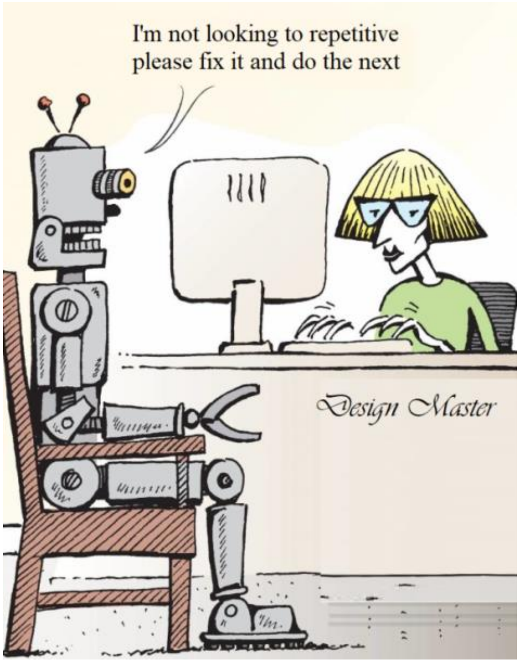

Fig. 1.3 Presentation of Repetition Issue of Design Work in Engineering

We need to remove it to avoid such said losses to all of us and all humanity. As per the key objective of science and technology, we need to work for all i.e. for the whole world, not for a limited area or some people. Therefore, it is must be made a unique and globalized world of design and modeling; by which we can save time and cost also, and enter to a beautiful and highly scientific and technical world, and also can enhance the economy of the world at a high level, not only of a country and hence, also can upgrade the living environments.

1.2.3 Technical Complications in case of complex objects

We encounter many kinds of complications during design, modeling, and analysis (DMAA) of a complex object, such as functional issue, various industrial issue, standardization, mechanical strength, and structured analysis, electrical capability, electronics complications, and control related issue, electromagnetic analysis, manufacturability, testability, output related issue, safety measures, various environmental conditions, economical issue, regulatory and statutory compliances, any other approval related issue, and the main thing is a funding issue, which comes under cost analysis, and also need to take in account one another important issue, i.e. time problem.

Fig. 1.4 Presentation of Complications Issue in Engineering

These kinds of many issues in the design and analysis of an object and after that, the setting of the life and many issues come during life of the object or product. Therefore it is an important aspect for a design engineer to solve this on an easy and clear way, that is a modern problem that looking to a new, easy and inexpensive platform or system, that can satisfy to all such issue in a click, within a second and then can show the easy way to design and modeling of any complex object.

1.2.4 Expensive Process of Design and Analysis

There are a lot of complications and various issues also discussed in the above section; to handle them, Today, available methods of design and modeling are meet too expensive, due to the long procedure, more time consuming and complex procedure, and also due to various testing and analysis requirements.

It is the prime facia of the family of issues related to engineering design, modeling, and analysis. As a design engineer, we are committed to resolving such cost issues and reducing the cost of design, modeling, and analysis (DMAA) procedures in various ways.

Fig. 1.5 Presentation of economic losses due to the looseness of engineering and improper use of technology

The work of this thesis presents some unique findings in that way of resolution to reduce the cost of engineering design, modeling, and analysis (E-DMAA).

1.3 Literature Survey and Review (LSR)

Further, in the way of resolving to discussed issues in the above section, we studied many papers, after setting our focus on some key and specific centroids; such as morphological, topological, geometrical issues, that consuming more cost and time during the design, modeling, and analysis (DMAA) of any complex objects. And, as discussed earlier in previous sections about the topic of the thesis in such a field, i.e. in Geometrical Design and Modeling (GDM), we present an advance study of Non-Uniform Rational Basis Spline Curves and Surfaces, and if see in-depth the subtitle is Superquadrics. We have studied more than a hundred papers including review articles, original research papers, technical notes, commentaries, editorials, and pictorial essays, by which we can find the better solution to such discussed issue of design engineering. The conclusion of such all these studies is summarized in section 2.1 of next chapter 2, in a sequence i.e. yearly and decade wise. For example in key papers there are, the seminal paper of Allen H. Barr, 1981, on ‘Superquadrics and its Angel Preserving Transformation’; His another paper of 1987, for ‘Global and Local deformations of Solid Primitives’; ‘Segmentation and Recovery of Superquadrics’ by Ales Jaklic et al; and ‘Superquadrics Revisited Learning 3D Shape Parsing beyond Cuboids’ by Despoina Paschalidou et al; and many others also are the key papers by whose study, this work and thesis are reflected [11].

We have done some case studies and shortlisted main points with a discussion respectively in section 2.1 of chapter 2.

1.4 Conclusion of Studies (CoS)

By this LSR we have an idea to the further work as in extension of the superquadrics, doing its geometrical modeling for various combinations of smaller and higher parameterization and also for various limit composition. And, finally, we are going on the same way as discussed also in the last paragraph of section 2.1 of chapter 2.

1.5 Base and Organization of Thesis (BOT)

1.5.1 Thesis Background

As per the MRI and LSR, we directed to plant an organization of operative thesis work based on superquadrics, it's mathematical modeling, geometrical modeling, extensions with the key objectives of improving the surface and solid modeling to optimize the design of engineering parts and products using computer graphics.

1.5.2 Curves and Surfaces

Here we presented a small introduction to the curves and surfaces to make a theme for the work, As we know that Curve is also known as Parametric curve and it divided in

1.5.2.1 Straight Line

The general equation of the straight line is as given below,

$$ax + by + c = 0$$

Where a, b, c are constants, and x, y are variables.

The relation between variables x, y satisfy all points on the line.

1.5.2.2 Quadratics Curves

A quadratic curve can be created by three distinct points such a curve is called a spline curve. In addition to a spline curve, a quadratic curve can be defined by two endpoint and a vector and by three control points forming a control polygon that encloses a Bezier curve among others

a) Spline Curve with three control points

The spline curve with three control point going through these three points one is start point and another belongs in the middle and last will be on the endpoint.

b) Spline curve with 2 control points and a vector

A quadratic curve can also be defined by its start and endpoint plus a vector, either at the start or the endpoint. Such a curve is known as a spline curve with 2 control points and a vector.

c) Bezier Curve

The Bezier curve, developed by Bezier in the year 1970, has become one of the most commonly used curves for geometric modeling. Bezier curve is defined by control points that do not necessarily stay on the curve. The control points form a control polygon (or characteristic polygon) that determines the shape of the curve. More specifically, in general, only the first and last control points stay on the curve; in fact, in this case, they coincide with the start and endpoint of the curve, respectively. The curve is also tangent to the first and last line segments of the control polygon, which provides the designer with direct control of the geometric shape of the curve at the ends. In addition to controlling the tangent vectors of the curves at ends, changing the control point locations alters the shape of the curve.

1.5.2.3 Cubic Curve

A cubic curve can be written in the following parametric form,

$$p(u) = [u^3 \quad u^2 \quad \mathrm{u} \quad 1] \begin{bmatrix} a_{3x} & a_{3y} & a_{3z} \\ a_{2x} & a_{2y} & a_{2z} \\ a_{1x} & a_{1y} & a_{1z} \\ a_{0x} & a_{0y} & a_{0z} \end{bmatrix} = U_{1x4}A_{4x3}$$

Where matrix A contains 12 (4x3) unknown coefficients

a) Spline Curve with four control points

A cubic curve with 4 distinct control points is known as a spline curve with four points.

b) Hermit Cubic Curve with two endpoint and two vector

Similar to the quadratic curve, a cubic curve can also be defined by its start and endpoint plus tangent vectors at the start and endpoint, which is called a hermit curve with two points and two vectors.

c) Cubic Bezier Curves

- They always pass through the first and last control points.
- They are contained in the convex hull of their defining control points.
- The degree of the polynomial defining the curve segment is one less than the number of defining polygon point.
- A Bezier curve generally follows the shape of the defining polygon.

1.5.2.4 Basis Spline Curves

A B-spline or basis spline is a spline function that has minimal support with respect to a given degree, smoothness, and domain partition. Any spline function of a given degree can be expressed as a linear combination of B-Splines of that degree.

d) Uniform Basis Spline Curves

Uniform cubic B-spline curves are based on the assumption that a nice curve corresponds to using cubic functions for each segment and constraining the points that joint the segments to meet three continuity requirements

e) Closed and uniform Basis Spline Curves

The curve shown above is an open B-spline curve, in which the start and end control points do not coincide. Uniform B-spline curves are well suited for modeling part geometry of a smooth closed profile. In this case, its control polygon must be closed, which can be achieved by simply aligning the first and the last control points.

1.5.2.5 Non-Uniform Rational Basis Spline Curves

Similarly, the family of surfaces is classified as parametric surface and B- Spline Surfaces, in Parametric surfaces there is 4 type of surfaces as listed below while in B- Spline surfaces only the B –Spline type is available.

1.5.2.6 Parametric Surfaces

A parametric surface is a surface in the Euclidean space R3, which is defined by parametric equations with two parameters. Parametric representation is probably the most general way to specify a surface. The curvature and arc length of curves on the surface, surface area, differential geometric invariants such as the first and second fundamental forms, Gaussian, mean, and principal curvatures can all be computed from a given parameterization. Due to their generality, parametric surfaces are widely adopted in geometric modeling for support of product design and manufacturing, among many other applications.

a) Bi-cubic surfaces patch

A bi-cubic surface patch can be defined in terms of cubic polynomials.

b) 16-Point Surface patch

A bi-cubic surface patch can be created by 16 distinct points arranged in a 4x4 matrix form.

c) Coons Patch

Coons patch (named after Steven Anson Coons, 1912–1979) is a bi-cubic parametric surface formed by four corner points, eight tangent vectors (two vectors in u and w directions, respectively, at each of the four corners), and four twister vectors at the respective four corner points.

d) Bezier Surface Patch

Bezier surfaces are a species of mathematical spline used in computer graphics, computer-aided design, and finite element modeling. As with the Bezier curve, a Bezier surface is defined by a set of control points.

Some other type of surface-based on geometry come such as,

e) Cylindrical Surfaces

As discussed earlier, in geometric modeling, a cylindrical surface can be considered as sweeping a straight line along a path curve. A cylinder has two-level or flat surfaces that are equal in surface area, coupled with a curved tubular (non-flat) surface through its height.

f) Ruled Surfaces

A ruled surface is defined by two path curves on the opposite sides of the surface, in which the trace of a straight line with its start and endpoint pass through the respective path curves with the same parametric value generates a ruled surface. A ruled surface can be described as the set of points swept by a moving straight line. For example, a cone is formed by keeping one point of a line fixed whilst moving another point along a circle. A surface is doubly ruled if through every one of its points there are two distinct lines that lie on the surface.

g) Loft or Blend Surfaces

When we loft a solid or surface feature using more than two sketch profiles, we generate a loft (or blend) surface, instead of a ruled surface.

h) Revolved Surfaces

When we sketch a profile and revolve it along an axis, the trace of the profile forms a revolved surface or surface of revolution

i) Sweep Surfaces

The trace of moving a profile curve along a path (or trajectory) curve is called a sweep surface.

1.5.3 Solid 3-D Design and Modeling

Basis methods for representing solid modeling are known as wireframe, surface, and solid forms. Another classification of solid modeling is of two types which are known as major modeling methods and named Constructive Solid Geometry (CSG) and other is boundary representation (B-Rep).

1.5.4 Computer Graphics and Modern Software

As we all know that today computer graphics become a part of our life in various ways and applications. For example, In Television, computers, Mobile phones and other many places we can see the use of computer graphics. Basically, it categorized into two categories one is Raster Graphics and the other is Vector Graphics. The list of some modern software for engineering design, modeling, and analysis kind of work, are given as below,

Table 1-1 List of Software

Name	Description
Advance Design	BIM Software for FEM structural analysis
ArchiCAD	BIM & 3D modeling software applied for civil & structural engineering
COMSOL Multiphysics	Simulation and multiphysics applied for structural engineering
Extreme Loading for Structures	Advanced non-linear structural analysis software

FEATool Multiphysics	Simulation and multiphysics applied for structural engineering
FEMtools	FEM software program providing advanced analysis and scripting solutions for structural engineering
FreeCAD	An open-source Swiss Army knife of general-purpose engineering toolkits
MathMod	For Geometrical modeling using Mathematical Script
Matlab	For Simulation and modeling
MicroStation	BIM & 3D modeling software applied for civil & structural engineering
OpenSees	Earthquake engineering software
Realsoft 3D	General 3D analysis and design software
Revit	BIM & 3D modeling software applied for civil & structural engineering
RFEM	3D structural analysis & design software
SDC Verifier	Structural verification and code-checking according to different industrial standards
SimScale	Multiphysics simulation (CFD, FEA, Thermal Analysis) applied for structural and civil engineering
SketchUp	BIM & 3D modeling software applied for civil & structural engineering
Tekla Structures	BIM & 3D modeling software for civil & structural engineers
Actcad	For Engineering Design and / or Modeling
AgiliCity Modelur	For Engineering Design and / or Modeling
Dassault Systemes CATIA	For Engineering Design and / or Modeling
Dassault Systemes SolidWorks	For Engineering Design and / or Modeling
Kubotek KeyCreator	For Engineering Design and / or Modeling
PTC PTC Creo (formerly known as Pro/ENGINEER)	For Engineering Design and / or Modeling
Siemens Solid Edge	For Engineering Design and / or Modeling
Trimble SketchUp	For Engineering Design and / or Modeling
Alibre Design	For Engineering Design and / or Modeling
AutoCAD	For Engineering Design and / or Modeling
Autodesk Inventor	For Engineering Design and / or Modeling
AxSTREAM	For Engineering Design and / or Modeling
Bentley Systems - MicroStation	For Engineering Design and / or Modeling
BricsCAD	For Engineering Design and / or Modeling
Cobalt	For Engineering Design and / or Modeling
Draftsight	For Engineering Design and / or Modeling

IRONCAD	For Engineering Design and / or Modeling
IntelliCAD	For Engineering Design and / or Modeling
MEDUSA	For Engineering Design and / or Modeling
Onshape	For Engineering Design and / or Modeling
ProgeCAD	For Engineering Design and / or Modeling
Promine	For Engineering Design and / or Modeling
PunchCAD	For Engineering Design and / or Modeling
Remo 3D	For Engineering Design and / or Modeling
Rhinoceros 3D	For Engineering Design and / or Modeling
RoutCad	For Engineering Design and / or Modeling
Siemens NX	For Engineering Design and / or Modeling
SketchUp	For Engineering Design and / or Modeling
SpaceClaim	For Engineering Design and / or Modeling
T-FLEX CAD	For Engineering Design and / or Modeling
TurboCAD	For Engineering Design and / or Modeling
VariCAD	For Engineering Design and / or Modeling
123D	For Engineering Design and / or Modeling
BRL-CAD	For Engineering Design and / or Modeling
BricsCAD Shape	For Engineering Design and / or Modeling
FreeCAD	For Engineering Design and / or Modeling
LibreCAD	For Engineering Design and / or Modeling
QCAD	For Engineering Design and / or Modeling
OpenSCAD	For Engineering Design and / or Modeling
SolveSpace	For Engineering Design and / or Modeling
Parasolid by Siemens	For Engineering Design and / or Modeling
ACIS by Spatial	For Engineering Design and / or Modeling
ShapeManager by Autodesk	For Engineering Design and / or Modeling
Open CASCADE Open Source	For Engineering Design and / or Modeling
C3D by C3D Labs	For Engineering Design and / or Modeling

1.5.5 Conclusion and Finalization of the Study Topic

In view of the above MRI, and advancement of NURBS and Superquadrics we decided to focus on such topics. And therefore, finalized to do the various study, testing and experiments based on such topic.

1.6 Non-Uniform Rational Basis Spline Curves and Surface (NURBS)

A parametric curve, capable to draw geometric entities with curvature, for example, a circle or any conic curves or their parts, is known as Non-Uniform Rational Basis Spline Curve (NURBS), which is one of the most versatile and general curves employed for geometric design and modeling.

The General Equation of NURBS are given below,

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,k}(u) W_i P_i}{\sum_{i=0}^{n} W_i N_{i,k}(u)}, \quad where\ u \in [0, (n+1) - (\mathrm{k}-1)]$$

And

$N_{i,k}(u)'^{s}$ *are the basis functions of the B − spline curve*

W_i *is the wieght function associated with* i^{th} *control points* P_i *and*

$n + 1$ *is the total number of control points* [7].

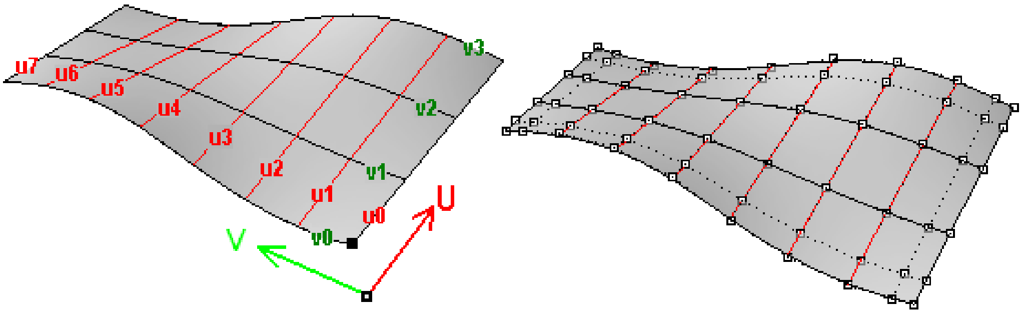

Fig. 1.6 An Example of NURBS

I have studied many kinds of works related to the geometry & graphics for example research /review papers, books, patents works, and other online resources. Where it is clear that there abroad works have been done in the past and currently a competitive environment can be seen in the design & development of curves and surfaces to achieve an enhanced quality of product and the lowest deviation between the virtual design and actual production using Computer Graphics.

Before computers, designs were drawn by hand on paper with various drafting tools. Rulers were used for straight lines, compasses for circles, and protractors for angles [9]. But many shapes, such as the freeform curve of a ship's bow, could not be drawn with these tools. Although such curves could be drawn freehand at the drafting board, shipbuilders often needed a life-size version that could not be done by hand. Such large drawings were done with the help of flexible strips of wood, called splines. The splines were held in place several predetermined points, called "ducks"; between the ducks, the elasticity of the spline material caused the strip to take the shape that minimized the energy of bending, thus creating the smoothest possible shape that fit the constraints. The shape could be tweaked by moving the ducks [103].

In 1946, mathematicians started studying the spline shape and derived the piecewise polynomial formula known as the spline curve or spline function. I. J. Schoenberg gave the spline function its name after its resemblance to the mechanical spline used by draftsmen [103].

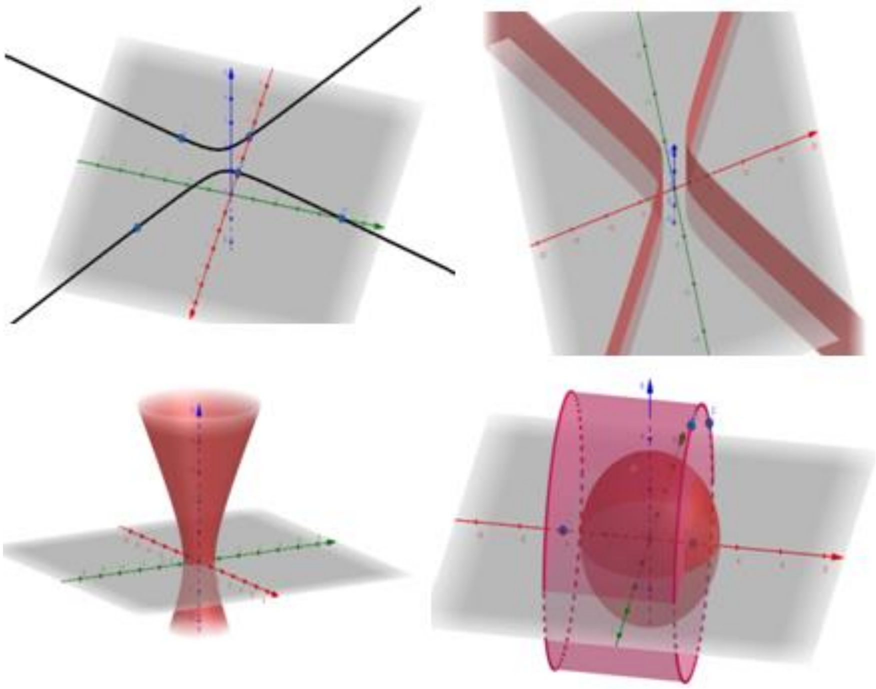

Fig. 1.7 Conic Curves and Surfaces designed on geogebra.org

As computers were introduced into the design process, the physical properties of such splines were investigated so that they could be modelled with mathematical precision and reproduced where needed. Pioneering work was done in France by Renault engineer Pierre Bezier, and Citroën's physicist and mathematician Paul de Casteljau. They worked nearly parallel to each other, but because Bezier published the results of his work, Bezier curves were named after him, while de Casteljau's name is only associated with related algorithms.

At first, NURBS were only used in the proprietary CAD packages of car companies. Later they became part of standard computer graphics packages.

Real-time, interactive rendering of NURBS curves and surfaces was first made commercially available on Silicon Graphics workstations in 1989. In 1993, the first interactive NURBS modeller for PCs, called NoRBS, was developed by CAS Berlin, a small startup company cooperating with the Technical University of Berlin.

Basic Importance that's why NURBS:-

Today, NURBS is widely used in computer-aided design due to its generality in the expression of curves and surfaces. Some reasons for choosing the NURBS are as follows:

1. They provide a common parametric form for designing, modelling and simulating both standard analytic shapes (Conic, cubic and quadratic) and free-form curves and surfaces.
2. By manipulating the control points as well as the weight function. NURBS are able to design a lot of shapes & sizes easily.

3. Evaluation is reasonably fast in different aspects and computationally stable for different kinds of designs.
4. NURBS have clear geometric interpretations, making them particularly useful for designers, who have a very good knowledge of geometry-especially descriptive geometry.
5. NURBS have a powerful geometric tool kit (knot insertion refinement removal. degree elevation, splitting, etc.), which can be used throughout to design, analyze, process, and interrogate objects.
6. NURBS are invariant under scaling, rotation. Translation and shear as well as parallel and perspective projection.
7. NURBS are genuine generalizations of non-rational B-spline forms as well as rational and non-rational Bezier curves and surfaces [102].

Since the proposal is based on the optimization of designs and modeling, it may to assume the hard and sincere working to achieve the list out objectives and the positive result of the whole work. I hope it will be useful for both industrial and academic along with enhancing my career also.

In the view of above, the proposed work is to enhance the knowledge and skills of designers, engineers, students, and all other people, who involved in the field of Science, Engineering, and Technology.

1.6.1 Superquadrics (Sq)

The superquadrics included many shapes that resemble cubes, octahedra, cylinders, lozenges, and spindles with rounded or sharp corners. Because of their flexibility and relative simplicity, they are popular geometric modelling tools, especially in computer graphics. Some basic examples of its geometric models are given below,

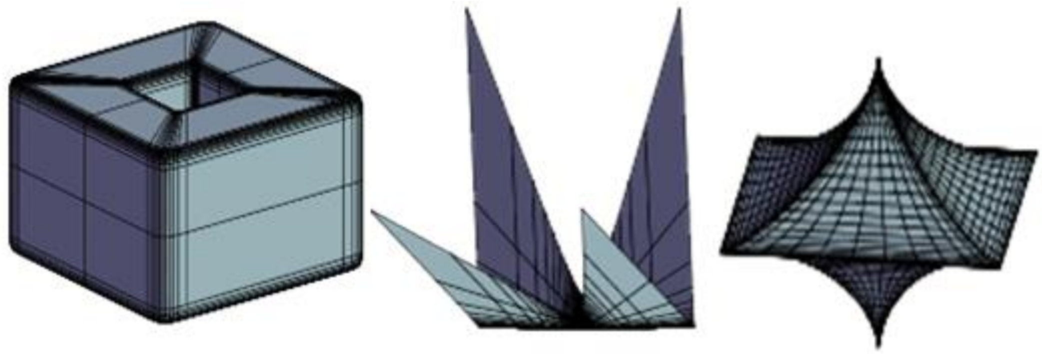

Fig. 1.8 Examples of Superquadrics

1.7 Whole Work Summary (WWS)

1.7.1 Aims and Objectives and Findings of the Work

Some basic and common objectives are listed out as below,

- To study the design, model & analyze the various parameters using Mathematical Modeling and Computer Graphics.

- ➢ To analyze the effects of various mechanisms, parameters, and relationships among them, which related to design and modelling of complex objects.
- ➢ To analyzing the quality of curve and surfaces, to reduce the possibility of failure and increase the strength and visualization as well as the product cost.

1.7.2 Advancement in Mathematical Preliminaries of Superquadrics

We presented the mathematical preliminaries for Superquadrics with third Exponent (Sq-WTE) along with their geometrical modeling, as the topic is discussed in detail in section 4.2.3 of chapter 4.

1.7.3 Developments of Geometrical Models Using Superquadrics

Using various limits and to optimize the limit range and for the parameterization of the higher value of exponent value combination of superquadrics and also for other. We use MathMod 9.1 and before it much other software such as MatLab, AutoCad, GeoGebra, GNU Octave (GUI), and many others for various kind of design, modeling, and analysis of models. As we achieved new models for the third exponent and airtight superquadrics model for the updated limit range. It is seen during the modeling of superquadrics that for higher parameterization i.e. for exponent value greater than 4 there we found models for only some combinations.

1.7.4 Applications, Advantages, and Limitation of Presented Models

Applications, advantages along with limitations are discussed in chapter 5 of Results and Discussion. As discussed in LSR for other superquadrics applications in various filed such as in Computer Graphics, Computer-aided Design, Primitives, Reconstruction, and in other design and modeling of a wide range of parts and products, the achieved extended superquadrics models also can be used as per requirement in all such applications. The advantages also are same as previous superquadrics as well as limitations.

1.7.5 Results, Discussions, Conclusions and Future Perspective

This is remarked by the results and discussion made on such results that the Mathematical Modeling is an easy way to represent various models, Grid Density, and Poly No. giving various unique models in case of superquadrics.

Here we found other work such as to summarise the models b checking for various remaining exponential combinations with third exponent, grid size, limits, and angular variation.

Tri Exponential and Gridded Superquadrics are found important and key finding of the work and also needs to investigate for some other models as per availability and also the mathematical explanation for limit error by considering tightness factor need to be draft and publish in future.

All these terms related to findings, results, conclusion along with future perspective are summarized in the last chapter 6 of conclusions and closing.

CHAPTER 2: REVIEW AND RELATED LITERATURE STUDY

2.1 Literature Survey and Review (LSR)

2.1.1 Before 1980

It was the 17th day of September month and the year was 1826 when a lady that was the wife of a Poor Lutheran Pastor birth a child, to that child, who going taught a new lesson of mathematics to the whole world. Yes, we are talking about the lady Charlotte Ebell, and her husband Friedrich Bernhard Riemann, who was a great parent of that child, who is the father of a large no. of mathematical theories, concepts, formulas, geometry, various relations, surfaces, solids and other many more things, not only in mathematics and physics but also in space science. Yes, I am talking about the great mathematician and scientist Georg Friedrich Bernhard Riemann. Who is the father of Riemannian geometry, Riemann bilinear relations, Riemann integral, Riemann surface, Riemann sphere, Riemannian cubic, Number theory, Free Riemann gas, Riemann tensor (general relativity), Riemannian symmetric space, Riemann's differential equation, Riemann's existence theorem, Riemann's explicit formula, Riemann's minimal surface, Riemann's theorem on removable singularities, Riemann (crater), Complex Analysis, Cauchy–Riemann equations and other many more contributions to Science and Technology [17]. That was the era of 3-D genesis. When, the mathematicians introduce many solutions for 3-D problems and there were produced many theories and phenomena related to 3-D geometry and 3-D object representation [19].

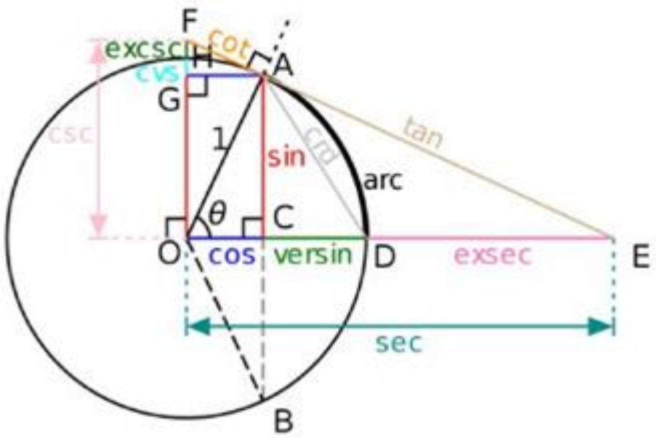

Fig. 2.1 An example of Euclid Geometry [24]

Let me tell you that, before it, in mathematics, in geometry, there were use only Euclidean Geometry. Which deals in 2-D only, as we have studied in high school, various theorems related to 2-D geometry. And, it also meets in various graduate studies that correlated plane geometry, algebra and number theory also. And, for this system, we attributed to the mathematician Euclid. Who contributed various methods of proof, a system of geometrical construction, and many other findings but only in

2-dimensional concepts, as it was the ancient period of 15^{th} to 16^{th} centuries; and then, there was the starting of geometry [18]. The Euclidean geometry is based on five postulates that are known as Euclid's postulates [23].

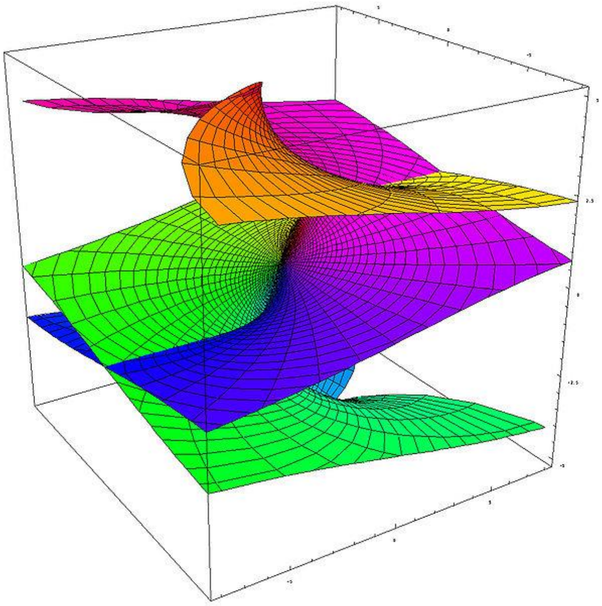

Fig. 2.2 Riemann Surface [25]

Now, let us return on our topic base, 3-D geometry i.e. Riemannian Geometry that was introduced in 1854 by Bernhard Riemann, Indeed, this was the advancement of 2-D geometry and after that, it is also known as the part of differential geometry. what was this geometry? Let we know about its some basic points, Actually, This branch of differential geometry studies about Riemannian manifolds with a Riemannian metric. Now you will say, two other terms, mean more complex to understand. But Don't worry, Let's understand, in details, In fact, Riemannian manifold is also known as Riemannian space, that is a real smooth surface equipped with a positive definite inner product Ip on the tangent space TqM at each point q; and since the phenomenon was introduced by Riemann, therefore, it is known as Riemannian manifold. That is the primary concept of 3-D object formation in mathematics. The said inner product is found in a metric form and that metric is also known as Riemannian metric or Riemannian metric tensor; and these terms makes it possible to define and distinguished about some base geometrical terms such as length of a curve, area of surface, the volume of solid, intrinsic and extrinsic curvature and many other terms of 3-D geometry of any object [19-22]. This Riemann geometry later became the fundamental concept of the theory of relativity introduced by the great scientist Albert Einstien. In such a way, 3-D Coordinate system, Cylindrical Coordinates, Spherical Coordinates, Regular Polygonal, Spline, Basis Spline, Uniform, Non-Uniform Spline, Curve and Surface Revolution, Bezier Surfaces, Non-Uniform Rational Basis Spline Curves and Surface, Quadrics Surfaces and then Superquadrics Surfaces had been introduced by various scientists, engineers and mathematicians before end of the year 1981. In such a regular development of various fields, such as geometry, design, modeling, and analysis, there were, became like a flood of techniques, knowledge, when computer developed. And in such a flood, In 1979, Faux, I. D., and Pratt, M.J. introduced to spline related composite curves and patched surfaces with the covering of both parametric and non–parametric techniques [17]. After that, in 1980, the thesis of JM Keil on computational geometry on integer grid presented an extended form of some advance structure such as cuboids by using a set of rectangles, and when two rectangles give polygons by intersecting to each other and many other concepts that look in the extension of surfaces and solid theory of mathematics along with making the base of computational geometry; Today, which has a hybrid, wide and complex form.

2.1.2 From 1981 to 1990

Because of advancement and as per findings of this study of various review and experiments and also seeing of future perspective, we set objectives, which correlates directly with superquadrics for present and future developments also in the fields of 3-D geometry and computer graphics, and hence we decided to focus on superquadrics only instead of wasting time on other available options. And, thus here, we discuss only on superquadrics in this and further sections also.

It was the year 1981 when the concept of superquadrics had been introduced by a doctoral student in Mathematics at Rensselaer Polytechnic Institute of New York, The Name of this student is Alan H Barr, As he stated that He investigated the mathematical modeling and simulation of mechanical and biological processes using computer animation to communicate time-dependent and 3-D outputs.

The mathematical preliminaries related to basic of superquadrics and its various branches had been published by him in his seminal paper of 1981 [1]. Where he has discussed the generation of 3-D solid by using a spherical product of two distinct curves which being discuss in chapter 3 of this thesis. Along with valid mathematical explanations and geometrical presentation of their models, their normal and position vectors, various position such as canonical position, general positions, transformations, angle preserving transformations, rotations, and various applications were discussed in that paper [1].

After that many more research become in light on this topic, such as in 1982, AR Smit has presented a parametric mapping of superquadrics assuming the spherical product surface [27] and in 1983 F Herbert had presented some primitives using a volume encoding data, which was a great finding in the field of solid modeling with encoding data instead of conventional edge-face-vertex for the presentation of 3-D objects. The structure of this volume encoding data is called as OCTREE in his paper [28]. After this, a new technique of computation presentation was presented by P.A. Koparkar and S.P. Mudur in 1984, wherein their paper, they explain the presented computational technique sing subdivision algorithms for the processing such as rendering, forming, and various operations between the solids e.g. addition, intersection, and subtraction. In the abstract of their paper of 1984, they write that they have done it by working in extrema of the visibility function and a class of surfaces is found by product and may be called product surfaces. In the paper, they describe Euclidean bounds, surface patches, the linearity of curves, and local visibility also [29]. In this line, a method for automatic mesh generation for 3-D solids and surfaces was published by Mark A. Yerry and Mark S. Shephard, where a full automatics element mesh generation was presented by them in their paper. This was only for finite element and the technique called modified octree technique. In the development of this technique, they have used the Computer-aided Design (CAD) for solid 3-D modeling. And, the used solid modeler in this work was based on Superquadrics Primitives [30].

An interesting application became in light when F Solina ad R Bajcsy presented a Superquadrics based model for modeling of mail pieces in the united state, they said in their paper of 1986, that the degradations from the ideal prototype are difficult to represent 3-D shape like generalized cylinders and polyhedral approximations and they proposed a model which was based on Superquadrics because of the roundness and squareness quality of surfaces and edges [31].

The recovery of superquadrics by depth information and also by 3-D information came in light in 1987 and 1988 respectively, These both concepts were presented by TE Boult and AD Gross, wherein first paper they discussed some rational solids using superquadrics for object recognition and also the said that superquadrics are sufficiently flexible for the representation of a broad class of solids and surfaces in its abstract, and also they said that both, its and its normal surfaces are well defined by inside and outside functions, that inside-outside functions works as a useful tool for the recovery of superquadrics. These insides-outside functions for various types of superquadrics are discussed in chapter 3. They presented a well quality of recovery in their paper using the least square minimization technique on such an inside-outside function. And the given result is shown that the inside-outside function allows both positive and negative instant points to recovered from depth data for the case of superquadrics [32]. In the second paper, they presented some examples by using both synthetic and actual range data, where the system recovered successfully for negative instant points of superquadrics. They also discussed a relationship between three different functions based on their inside-outside functions [33].

Another characterization of Darboux frames and some other solids were presented using superquadrics by FP Ferrie et al in the year 1989, where they derived some 3-D articulated volumetric description of these solid objects from a laser range finder. This representation of computation models shows an interpolation between features and described local surfaces. They used superquadrics subsequently to characterize 3-D solids and surfaces and also its partitions [34].

After this in 1990, F Solina and R Bajcsy again published a paper for the recovery of compact volumetric solids and surfaces. The presented models were based on parametric deformations such as bending, twisting, tapering, and deformation of the cavity. They provide 3-D range inputs to the system and by iterative gradient and descent minimization process were models adjusted simultaneously as they presented in the paper that means the recovery position, orientation, size and shape of solids and surfaces and other parameters were getting adjusted. They were also written that the recovered models were stable and the procedure was so fast when using such real range data [35].

2.1.3 From 1991 to 2000

In this line of development of Superquadrics famous article which was published in 1991 by IEEE Transactions on Pattern Analysis and Machine Intelligence written by D. Terzopoulos and D. Metaxas, in which, dynamic 3-D models with local and global deformations were introduced and that was the first time when deformable superquadrics along with its dynamic applications were introduced. In this article, the authors presented a solid based approach for the fitting of complex 3-D shapes and solids by using a new family of dynamic models. All the members of this family can deform in both statuses that are globally and locally, as we have said that such kind of deformations was introduced before this time and now the concept moves to a dynamic world from static. Here the presented mathematical equations for the motions that govern the behavior of solids and surfaces, the superquadrics make them responsible for externally applied forces. The said model was seeing fit for visual data by transforming the data into forces and simulating the model of mathematical equations. The degree of freedom for translation, deformation, and rotation there adjusted. And their experiments were done for 2-D monocular image data and also for 3-D range data as presented by him their paper [36]. Another some models such as constrained deformable superquadrics by D Metaxas and D Terzopoulous also was published in the same year [37], Superquadrics for 3-D shape indexing language by T Horikoshi and H Kasahara gives the details of input parameters, presentation methods and processing technique for 3-D images for the improvement of the quality aspect of backgrounds [38] and some other paper were published for superquadrics 3-D dynamic models.

In the next year 1992, JR Williams and AP Pentland presented an advanced study in interactive discrete element simulation for a computer-aided design using superquadrics. They lighted up some computational problems for the virtual world of solids and surfaces. By using coupling between superquadrics models they achieved two orders of magnitude in the efficiency of physical dynamic models as well as virtual models. The presented method allows us to trade off high order models for improved stability and their beliefs in virtual manufacturing systems are in the form of reality in front of our eye site [39]. In the same year, NS Raja and AK Jain recognized some geons by using superquadrics which were fitted to range data. They presented that the classes of superquadrics are corresponding to a collapsed set of 36 different geons. They defined a classification error rates for nearest neighbor between 12 shape classes, that are based on some geometrical attributes such as axis type, cross-sectional edge, and variation of their sizes along with axis, that may as constant, tapered, increasing or decreasing behavior. In such a way, they obtain around 80% reliability for shape properties with a binary tree classifier [40]. It was cleared that Superellipsoids, Superhyperboloids, and Supertorids found so useful in computer graphics and in this line Alan AH Barr become in light with his rigid physical-based superquadrics, And this time he was introducing us with mathematical equations for calculation of the motion of rigid models along with their closed form of algebraic expressions for their center of mass, volume, inertial forces, and rotational forces. In this paper, he correlates with Newtonian physics of rigid body motion. He also tells us here that the superquadrics are a 3-D extension of superellipse introduced by Piet Hein. And, they allow us to easily presenting square, cylindrical, rounded, pinched, toroidal and rounded shape and it serves by some relatively simple mathematical equations. Some nonzero components of inertial tensors were also introduced by him in this paper for superellipsoids [41].

An integral framework for segmentation of dense range data of complex 3-D objects into the 2-D surface and 3-D volumetric primitives (also called superquadrics) were presented without prior domain knowledge of stored models by A Gupta, R Bajcsy [42]. Here their results were found on real images of some scenes of varying complications, and they said that their 2-D segmentation is not enough to direct the 3-D segmentation [42].

A representational computational model of Superquadrics for simulation of telemanipulation system with object interaction was introduced by EI Agba et al in the year 1993. Where supervisory control, training of operator and human factors and their performance was evaluated and developed. The presented simulator was found capable of operational control using superquadrics. It has a conventional inside-outside function for interacting and feature of object detection was also used to provide virtual performance in their experiments [43]. A free form deformation for medical images with their experimental results based on the presented model of Sederberg and Parry in 1986 was given by E. Bardinet et al in 1994 [44]. By applying the genetic algorithms also said an optimized technique, H Saito and N Tsunashima estimated some parameters from some shading images of various 3-D surfaces. These parameters were founded helpful in 3-D modeling of various solids and shapes using superquadrics as they correlate with genetic algorithms [45]. By using ray tracing and image synthesis algorithms for realistic photo presentation a different model was presented by H Loffelmann and E Groller in 1995 [46] and another 3-D model for the reconstruction of 3-D models based on superquadrics and basis spline parametric curves was presented by M Neveu et al in the same year [47]. The concept of recovery of hyperquadrics from range data also given by S Kumar et all for the field of robotics and computer vision [48].

Ratioquadrics as an extension of 3-D surfaces and a new family of 2-D curves called as ratioconics was presented by C Blanc and C Schlick in 1996. It was an alternative to the known superconics and also said for superquadrics but due to its various complications and disadvantages, it may not use in some cases of 3-D modeling [49]. By using unsegmented data superquadrics were recovered directly by A Leonardis et al in 1997. Where they present a method that is reliable and efficient for the recovery of part-descriptions in case of superquadrics models. The method was based on the recovery and selection of paradigm. With some useful examples of experiments on real and synthetic images, they were demonstrated the stability of the method [50]. For the measures of such recovered models, a similarity measure was proposed by L H Chen et al. Those measures were to evaluate the degree of shape similarity by distinguishing between two superquadric models [51]. To the analysis of the motion of such recovered models, a computer model was presented by F Marzani et al. They used motion analysis equipment to analyze the disabilities that appeared in movement. Their approach allows us to analyze human motion in 3-D space without using the tracing markers [52].

This was 1998 when H Zha et al presented a recursive fitting and splitting algorithm for solid modeling by using superquadric models. Where they said the given sensor data was a set of multi-view range data. And this data was covering the complete surface of the model. By starting from an initial approximation of the model the method goes to fitting and splitting of the model [53]. In view of the above, superquadrics had been taken as an important part of computer graphics and vision also. And, in this regard, a surface subdivision model for the generation of superquadrics was proposed by ME Montiel et al in 1998. This model resides the parameterized nature and this allows a wide range of solids and surfaces based on the part level description [54].

Shape description had been made the main research theme during that period of 1998-99. And in 1999, a new method for shape description of two superquadrics by using range data and genetic algorithms was proposed again by H Tanahashi et al. In that method two superquadrics were evolved for making a description of a 3-D shape [55]. These 3D shapes were extended by M Pilu et al in their paper of 1999. In this paper, they described a PDM model that means a point distribution model, in which they described deformable superellipses using Eigen models and their various fitting [56]. There were most conventional researchers, who use superquadrics because of its advantages and capability to describe a broad range of primitives [57]. And in view of such advantages and applications of superquadrics, the river of knowledge related to superquadrics was getting wider from day today.

In the line of the above, various geometric properties and applications of superquadrics were discussed by A Jaklic et al in their papers Superquadrics and their geometric properties; and Applications of Superquadrics published in 2000, wherein first paper they derived explicit equations and inertial moment mathematically and in the second paper along with important applications they highlighted some limitations also such as limited shape vocabulary and the fact check for really coarse grain for closed 3-D models. They said that like any other shape primitives, superquadrics may not represent arbitrarily. For example, bifurcating elongated models i.e. blood vessels can be described by a spine function along with a cross-section function but it not suitably described by Superquadrics. They also were giving a coordinate frame for superquadrics along with the extension of superquadrics in another paper published with the title extension of superquadrics; they also work on segmentation and recovery of individual superquadrics. Where they proposed a new criterion for the selection of geometric primitives and recognition application for scene description [58-62].

Along with these works in the same year, A Bosnjak et al presented a comparative study of some methods for ventricle segmentation and echocardiographic presentation using superquadrics where they developed some important images of echocardiographic data. Their project developed by four basic models in which 3-D reconstruction, segmentation, visualization, and acquisition were combined [63]. The quantitative analysis of such processes is important and for this analysis, a method was presented by V Torrealba et al. Where they analyzed the ventricle mobility, that was based on 3-D deformable dynamic models i.e. deformable superquadric models [64].

2.1.4 From 2001 to 2010

Robust and compact representation of superquadrics named by extending superquadrics with exponent functions was presented in 2001 by L Zhou and C Kambhamettu. They represented modeling and reconstruction of these extended superquadrics with statements for limitation of superquadrics such as intrinsically the symmetry for some real and natural models was failed in case of superquadrics use e.g. human body, animals and other naturally occurring objects. They consider longitude and latitude angle in a spherical coordinate system to explain the presented extension and claims that the extended superquadrics can model complex shapes easily compare that previously available superquadrics. They introduced with a new set of mathematical equations and some example of geometrical modeling of these equations where they intended to prove that the presented models and explanation is better than the previous model, some models among those presented are related to natural animal' body and some household articles [65].

In this way, geometric tolerance plays a big role during various Boolean operations between the models of superquadrics. And, this geometric tolerance was described by CC Barcenas et al, where they developed some standards for hard gage technology and answered that how tolerances can be verified for flexible technologies for example in laser scanners and measuring machines. In their study, they use a statistical technique using jackknifing for the verification of geometric tolerance. The technique presented various measurements for superquadrics parametric models. But also can apply in other representations of parametric models [66]. On the other hand, L Chevalier et al were presenting a geometric coding for the representation of virtual object transmission and three dimensional real objects also. They used Constructive Solid geometry along with superquadrics to describe the surfaces of such objects. Unstructured clouds of some points in 3-D space were also considered by them on the original object surface as initial data. Their algorithm allowed the merging of both connected and non-connected elements of superellipsoids. These superellipsoids formed by the decomposition of synthetic objects [67].

Further in 2002, a neural architecture for the segmentation and geometrical modeling of range data with un-deformed superquadrics was presented by A Chella and R Pirrone [68]. And in the same year, generalized Minkowski Metrics was proposed correlated with superquadrics by J Gielis in a technical report. The Minkowski Metrics is able to represent a family of complex solids and surfaces as well as tensor data and also apply for their boolean operation by Minkowski Sum and other operations [69].

After this in 2003, J Gielis et al came up with their explanation of rational and irrational symmetry. They introduced hyperspheres for CAD at the level of graphics kernels. They claimed that their approach presented an elegant way for both solid modeling and boundary representation when using 3-D geometry [70]. On the other side, Y Zhang et al was presenting a new method of presentation which was based on superquadrics for automotive parts. The method was started from a 3-D water-tight surface and ended to segmented original objects [71]. The segmentation and recognition of various objects were doing by using superquadrics. A part level object recognition [72] was presented that was reconstructed by range images. Also for the case of segmentation of the left ventricle was analyzed by measuring the temporal change of the spatial distribution of radioactive tracer. This phenomenon was presented by R Pohle et al in 2004. In this phenomenon, they were used free form of deformed superquadrics. And they achieved a good ascertain between the manual and automatic segmentation for all tested images [73]. A Computed tomography simulation was proved accurate and efficient for x-ray transmission using superquadrics. In medical use, X-Ray and CT scan are important and for the evaluation and improvement of their images, there was proposed an algorithm by computation of x-ray transforms using superellipsoids and tori. In this demonstration, they have used a monochromatic x-ray. There was stated that the presented model provides more realistic images compare to quadratic models and fast computation than a spline method [74]. Also for the shape recovery of such medical images, a method was introduced by T Bhabhrawala and V Krovi. They claimed that the presented method can improve the representation of the reconstruction of geometric shapes rapidly. They were presented an abstraction model as a development of a class of parametric shapes and said that as extended superquadrics. They applied that models for the biomedical and life science arena whose diversity and irregularities were found more difficult to understand and also difficult to clarify by using classical techniques [75]. Also, an efficient algorithm was proposed for contact resolution of 2-dimensional images in which any object approximated with a convex polygon through the adoptive method of sampling and after that using clipping of two polygons and then can perform the intersection of them or overlapping. The contact forces and directions were determined by specifying the corners of objects and this method also can apply on non-circular discrete models [76].

Further, in 2007, An object detection method was introduced for 3-D models by the fitting of superquadrics, It was for the robotics and the method was based on the Random sample consensus (RANSAC), which is an iterative method to estimate important parameters related to the object performance. In this method, the probabilistic search starts from a resolution of low quality and goes to higher values. Presented experiments demonstrate the effectiveness along with robustness that was found useful for robot grasp planning [77].

Using Constructive Solid Geometry (CSG) the rendering can be done and on this basis, TE Kim has presented a new method for reconstruction of 3-D shapes using models of superquadrics. He was assuming 11 parameters of superquadrics and also use the both un-deformed and deformed superquadrics. By using of 1-buffer algorithm and stencil buffer he has presented the synthesis process of 3-D models [78]. Equal distance Sampling is also important for such kind of rendering of 3-D solids when CSG but also in case of cost function optimization [79]. For the surface rendering, there was introduced the hyperquadrics as an efficient method of surface representation and its rendering for 2-D images and also for solid surfaces in computer graphics [80].

The point clouds concept in segmentation and approximation of these surfaces and also for solids becomes useful. For both, a solution-based method using superquadrics was presented by M Strand and R Dillmann in 2009. In which they proposed a surface and shape fitting method by assuming that all the points on the unique surface or shape belong to a set [81]. In this way in 2010, for novelty detection and to derive the multi-scaled sampling of superquadrics for 3-D shape retrieval, P Drews et al presented an extended form of Gaussian mixture models for the fitting of superquadrics and to detect the changes. The refinement and optimization were recorded by using the split and merge paradigm [82].

2.1.5 From 2011 to 2020

It was 2011 and the time was developing of articulated human models (AHM) by using various superquadrics. The geometric accuracy of the body shape to be improved and it was possible only by the improvement of superquadrics surfaces and solids. In this way a model was presented in this year by HM Lee et al presented a 3-step method for representation of AHM; In the first step, they said to divide the scanned human body into 17 segments as per the joints, in the second step, fitting is engaged to minimize the Euclidean distance between such divided segments and superquadrics. And in the last, the third step said to use the free form demonstration to improve accuracy over superquadric fitting [83]. Polyhedrons based on computational methods with superquadrics may also be seen useful in geometrical modeling and constructions work when designing the model using computer-aided design for sculpture design [84]. In such kind of representation, estimation for the various pose of the human body was made by using superquadrics in 2012 by IIya Afanasyev et al. In the presented method they used a multi-view camera system and segmentation by a special pre-processing algorithm. The said algorithm was based on clothing analysis and the model was built by using 9 superquadrics models. The said method was said to may use for pose estimation, recognition, and also for localization of views and parts of the human body [85].

In 2013, a solid object localization algorithm based on superquadrics was published, in which, pose parameters using unorganized point clouds were estimated for localization algorithms along with the normalized form of radial Euclidean distance in a 3-D space [86]. For like these cases another method of the 3-D gesture was proposed by IIya Afansyev and Mariolino Decco. This localization method is based on processing solidification data and color gloves acquired by a 3-D sensor. The result was verified by evaluating the matching score means that the number of inliers, distance from the model surface, and control points in 3-D space [87].

Further in 2014, a simultaneous segmentation method becomes in light for superquadrics fitting when using laser range data. In the proposed method said useful in the field of robotics, autonomous vehicles, 3-D modeling, namely object detection, and segmentation, In 2-D experiments the data collected from indoor and outdoor environments. And that data was analyzed qualitatively and quantitatively, also 3-D data obtained from an in house setup [88]. In such a way to treat breast cancer and the representation of these states to help in the investigation, the fitting of 3d point clouds was used in surgery planning to obtain promising results with real data. The method based on the fitting of point clouds to a parametric model of the breast [89].

By using Convex Superquadrics, second-order cone programming based on proximity query was proposed, in which proximity had been used broadly in the planning of robot trajectory, 3-D printing, automatic assembling, and virtual surgery also [90]. These discussed applications refer to mixed surfaces application and for such mixed surfaces, B Bulca and K Arslan proposed a new model based on 3-D Euclidean space, These superquadrics shown in fi. 2.3, were formed by product surfaces between the plane curve and space curve that product is also called spherical product which is discussed in the next chapter.

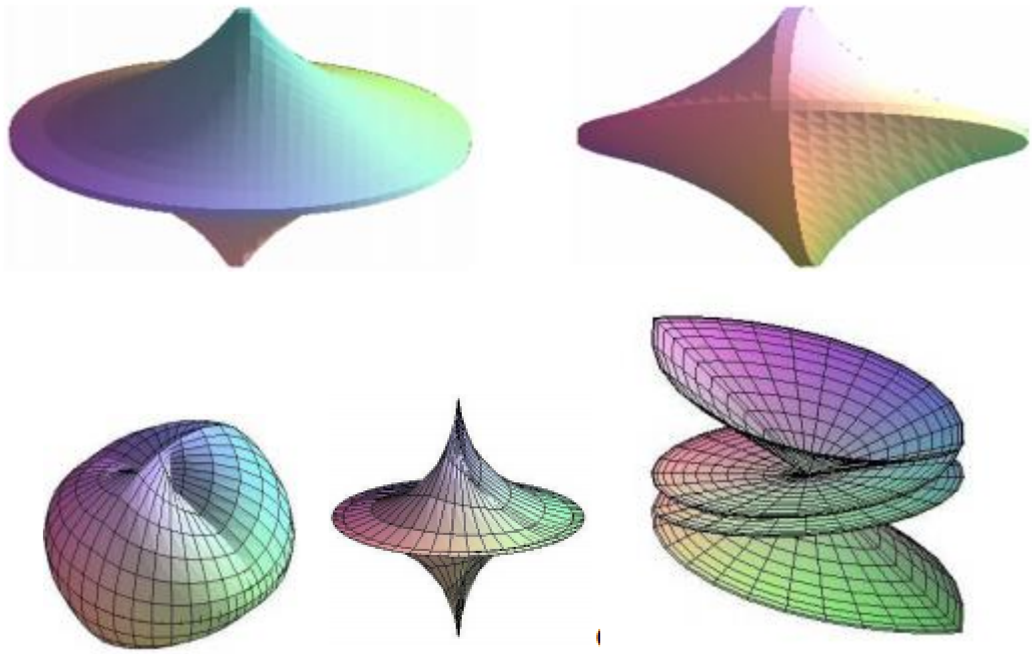

Fig. 2.3 Examples of Mixed Surface Superquadrics [91]

These Superquadrics are mathematically very simple and have the capability to obtain a range of solids and surfaces by using low order parameterization. Further, if we look in deep then the mathematical equations are of closed-form and that means they also can be used in the application of robotic movements. Such as in Obstacle, grasping, and avoidance. Volumetric representation with a mathematical explanation was presented for superellipsoids in the discussed work [92].

Recognition and localization of various sacks in racks uploading by using the fitting phenomenon of superquadrics for the autonomous container were introduced by N Vaskevicius in 2017, for the development of robotics using a low-cost sensor. The presented method makes a perception of the new robotic platform. that are embedded for manipulation of 70 Kg sacks when transporting various foods. The presented model was designed for the application of real-world scenarios [93]. For the extension of application for such a part of this real-world, various kinds of primitives were introduced and they make easy to design, modeling, planning, and analysis of various solid tasks and cases related to various fields. A large margin Nearest Neighbor Classifier (LMNN) for superquadrics parameters was recognized for shape authentication. These parameters also may use for vector classification. The proposed method in the paper of Ryo Hachiuma et al used these LMNN for recognition based on superquadrics. Where they also compare to previous methods and found a positive result for this method by used nearest neighbors 76.5% and support vector margin 73.5%, the performance of LMNN method was recorded by them are 79.5% accurate [94]. Both the multi-sphere and superquadrics have the options for adjustment in the geometry of particles to address the shape complexities such as for clamped spheres, controlling the number of possible sub sphere leads to the surface bumpiness. Changing the edge sharpness or can say blockiness of superquadrics can regulate by the angularity of the particles of the surfaces or solids [95].

Such Superquadrics also can be constructed by using classical Bezier Curves and also by using T-Bezier curves. Where this new curve has discovered recently and here T referred to trigonometric as this model is presented in 2018 by Piscoran et al [96]. And due to the use of curves for forming of superquadrics reconstruction of 3-D models was seeing easy compare that point clouds. Because Reconstruction or Sampling of Superquadrics using point clouds was tested at many times. For a new example in this year 2018, Paulo Ferreira came up with their paper for such sampling. Where he wrote to deal with the superellipsoid and superparaboloids because of non-linearity behavior, therefore it has clear that with curve modeling the sampling will be easy compare that point clouds. Whatsoever, Paulo Ferreira used parametric formulation for such sampling using point clouds with normal of superquadrics model. He made his approach close to the uniform system of sampling for superellipsoids and one single frame for reaming superparaboloids [97].

Further, in the next year 2019, Superquadrics revisited models with various examples were introduced by D Paschalidou et al which has well described in the next chapter and also that may use to understand the various application of superquadrics in mechanical engineering [11]. Except that to understand and control the various geometrical functions of autonomous machinery with their environment using computer graphics by 3-D virtual physical space in terms of parameterized virtual solids and surfaces or can say the superquadrics. A Conventional Neural Networks (CNNs) was proposed by Tim Oblak et al, for deep learning models using range images data [98]. An augmented rapidly and random exploring tree of superquadrics was presented in the thesis of [EFREM AFEWORK, 2019], where the thesis investigated about collision checking for rapidly-exploring random trees (RRT), advantages and disadvantages of superquadrics, motion planning for a higher degree of freedom, proximity querying along with saying that the collision detection is the main work of the thesis. Where, for the collision detection, they used GJK algorithm for compression with superquadrics. And, the conclusion of the thesis said that the superquadrics algorithm and RRT algorithm can be merged [99]. There are many other recent studies found in the year 2019. But due to a limited time period, here I have taken only some important and top listed in the account.

As we have discussed above the collision, the collision avoidance modeling varies for non-optimal, deterministic methods to optimal and probabilistic methods. The challenging multiple intruder collision avoidance problems (CMICAP) getting solved by the proposed model and it concluded that the superquadrics can be used to produce optimal trajectories by enforcing to multi-intruder dynamic and keep out to the regional constraints when receding horizon scenario; and the superquadrics can afford collision avoidance as per their great flexibilities to tailor unique keep out the regions, that optimize avoidance trajectories in a stochastic surrounding [100]. A generalization of superquadrics in 3-D Space for conic sections has introduced, where each conic section has the planner intersection on its quadrics surfaces. As shown in fig 2.4, a conoid appears for generalization of quadrics. It has two parametric families of the ellipse. That cut out from cylinders and that cylinder contains conoid's linear directrix.

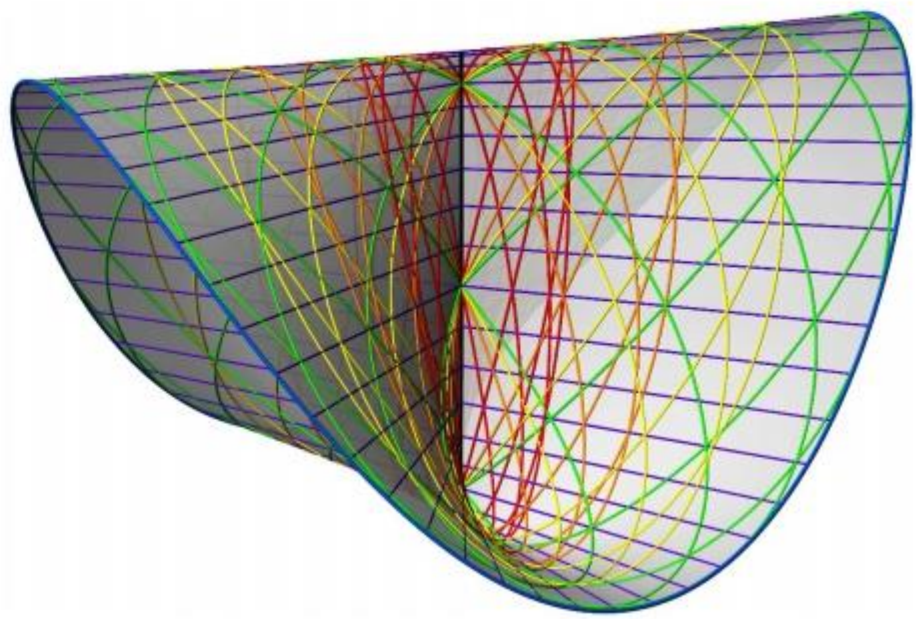

Fig. 2.4 A generalization of Superquadrics by curves reparameterization [100]

From the shape net dataset and on the D-Faust dataset, an additional method has tested in the supplementary document of D Paschalidou et al, which is earlier has published in this year 2020, They derived an occupancy function in Euclidean space R3 and found like points clouds for superellipsoids by considering three component of size parameters as well as before, and represented implicit function for the surface of superellipsoid. A feature encoder, which depends on the type of the inputs, such as binary input and an image like input. They make some partition using feature encoder architecture and that partition is termed as partition network, structural network, and geometrical network. The proximity loss also examines by using geometrical modeling and giving some examples such as the motion of the human body, on giving various inner forces as input and the deflected part due to such movement and hence registered loss due to proximity are presented clearly. Additionally, Reconstructions of various objects using super quadrics and proximity are discussed there [101].

In viewing of the above year, wise sequential Literature Study and Reviews (LSR) We have to find many latest points on which, we may work such as verification of geometric modeling of superquadrics for higher reparameterization, modeling for various limits, suppressing the various features of superquadrics, and also checking the validity of the same kind of equations and for their geometric models using modern computer graphics software for mathematical modeling. For example, by using other terms or parameters in the mathematical equations of superquadrics, we check for the possible valid extension of superquadrics.

CHAPTER 3: FEATURES OF SUPERQUADRICS

3.1 Mathematical Preliminaries of Superquadrics (MPSq)

The superquadrics (also known as superquadratics super-quadrics or super quadrics) are a set of 3D geometrical solids and surfaces achieved by the mathematical equations of superellipsoids, superhyperboloids, and supertoroids [1]. The superquadrics also can be called out as the nuclear family of these three subfamilies of superellipsoids, superhyperboloids, and supertoroids. The term "superquadrics" was first time used by Alan H. Barr in his seminal paper in 1981 and refers to solids and surfaces formed by arbitrary values of exponents in superquadrics equations [5]. In such families, it includes cubes, cylinders, octahedra, spindles and lozenges, and many other shapes with rounded and sharp corners. Due to flexible operation, simplicity, and lots of availability of shapes and solids, they are popular and trending geometrical modeling tools, especially in computer graphics [9].

Superquadrics show a generalization of all the basic quadric solids and surfaces and that resulted as a continuum of useful primitives of both rounded and sharp edges along with fillet faces. Angular-preserving transformations [1, 4], bending twisting and tapering [10] of such primitives [14] convert the object into a new form. Hence, together, these new forms of objects and primitives [10] have potential design applications [11] where flexible operations are available, and the volume, surface area, and/or arc length meets be conserved. In this way, Superquadrics provides a powerful enhancement to the classical design shapes.

The representation of 3D complex objects [6] that can be made in parts using different kinds of primitives is the trending strategy and such primitives are given by the Superquadrics family [3]. In computer graphics [9] Superquadrics are among the most useful primitive model [11] which may use on a large scale in various fields of design engineering. As it has a family of parametric models that cover a wide variety of smoothly changing 3D symmetrical shapes [13], which may be controlled by a few numbers of variables and can be increased by using Boolean operations upon global and local coordinates [2] e.g. union, intersection and subtraction. Thus, the superquadrics have a large range of applications in the design, modeling, and reconstruction of complex objects [3] with having much lesser numbers of variables than meshes [11].

After this many other explanations came out in the light as Blob model presented by Blinn in 1982, Hyperquadrics by Hanson in 1988, Gaussian images and its extended model represented by Horn in 1986, Symmetry seeking models represented by Terzopoulos in 1988, Spherical harmonic surfaces by Pentland and sclaroff in 1991, Implicit fourth-degree polynomials by keren in 1994, and as per max lame theory of superellipse by Ales Jakic in 2000 [5].

These all explanation is near to Alan H. Barr' theory and correlated to it. Blob model is based on a scalar field and produced by several filed primitives. The basic idea relates to some control points in space as the source of the potential area and then computes the 3D iso-surface of such an area to gives a threshold value [14]. In this model, Blinn originally used spherical fields but after those superquadrics filed were used by Wyvill in 1989. However, The Blob model was found computationally rather expensive.

Hyperquadrics show the family of superquadrics is a special case of hyperquadrics and it shows a generalized form of superquadrics with larger shape variability. As per this method, the superquadrics can be generated by taking hyperslices of high dimensional algebraic hypersurfaces as well as ratioquadrics. They all are found similar to superquadrics. The general equation of hyperquadrics is given as below,

$$H(x) = \sum_{a=1}^{N} (\sigma_a |H_a(x)|^{\gamma a} = C$$

Which is a linear combination of $N \leq n$ linear, H_a (x) is an independent function of the form,

$$H_a(x) = \left(\sum_{i=1}^{n} r_{ai} x_i + d_a) \right)$$

Where x is an n-dimensional vector while r_{ai} and d_a be the constants. As per our interest, they can be adjusted in 2D or 3D using hyperslices of these high-dimensional algebraic hypersurfaces.

Ratioquadrics are alternate moves around the superquadrics presented by Blanc and Schlick in 1996. They claim the finding of a smooth surface compares to superquadrics. The general explicit equations of ratioquadrics are given as,

$$r(\eta, \omega) = \begin{bmatrix} a_1 \dfrac{\cos\eta}{\varepsilon_1 + (1-\varepsilon_1)|\cos\eta|} \dfrac{|\cos\omega|}{\varepsilon_2 + (1-\varepsilon_2)|\cos\omega|} \\ a_2 \dfrac{|\cos\eta|}{\varepsilon_1 + (1-\varepsilon_1)|\cos\eta|} \dfrac{|cos\omega|}{\varepsilon_2 + (1-\varepsilon_2)|\cos\omega|} \\ a_3 \dfrac{|\sin\omega|}{\varepsilon_1 + (1-\varepsilon_1)|\sin\omega|} \end{bmatrix}$$

$$\text{where,} \quad -\frac{\pi}{2} \leq \eta \leq \frac{\pi}{2} \text{ and } -\pi \leq \omega < \pi$$

But the very small and very large value of ε_1 and ε_2 creates numerical imprecision which results in problems in graphical applications.

Gaussian images method shows the mapping of normal on a given surface. According to its extended form, the mass density is proportional to the surface area when the object is mapped over the unit area. This extension was made for convex shape [5]. The Gaussian images model was represented by Horn in 1986, and it’s extended form by Kang and Ikeuchi in 1993 and after this, the spherical representation was made by Delingette in 1993 [5].

The symmetry seeking model was represented by Terzopoulos in 1988, which is a generalization of cylinders and control using elastic deformation parameters [2] along the axis and walls of the cylinder [10].

Spherical harmonic surfaces methodology based on a 3D analogy of Fourier components. In which the low order harmonic coefficients represent gross shape style and the higher-order coefficients give a spatial frequency of the geometry. The superquadrics model accounted for the global degree of freedom [13] while this spherical harmonic surface model accounted for local degrees [12]. The main disadvantage of this methodology is that the individual parameter affects the whole geometry [5].

Implicit fourth-degree polynomials represent similar shapes just like superquadrics, which depends on the fitting of implicit polynomials and this method was given by Keren in 1994.

Equation of superellipse [5] is a family of quadratic curves well known as a part of Lame Curves in analytical geometry. Lame Curves was introduced by Gabriel Lame in 1818 [5]. A superellipse is a closed curve defined as,

$$\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 1$$

Where a and b are +ve real numbers and represents to the size of superellipse, well known as major and minor axes respectively, and m is a rational number uses as a single exponent, i.e.

$$m = \frac{p}{q} > 0,$$

Where p is even and q is an odd +ve integer.

As shown in Fig 2, If we take m = 4, it gives rectangle with fillet corner and if take m = 2 gives circle and for higher values of e.g. m = 50, it gives rectangle with sharp corner and for very small values of m, it gives just a part of curve or edges of a star [5].

In the topology, there are nine types of Lame curves due to the different values of m, here m is defined by Loria, IN 1910 as below,

Lame curves for +ve m, Lame Curves for –ve m,

$$\mathrm{m} = \frac{2\mathrm{h}}{2\mathrm{k}+1} > 1 \qquad \mathrm{m} = -\frac{2\mathrm{h}}{2\mathrm{k}+1}$$

$$\mathrm{m} = \frac{2\mathrm{h}}{2\mathrm{k}+1} < 1 \qquad \mathrm{m} = -\frac{2\mathrm{h}+1}{2\mathrm{k}}$$

$$\mathrm{m} = \frac{2\mathrm{h}+1}{2\mathrm{k}} > 1 \qquad \mathrm{m} = -\frac{2\mathrm{h}+1}{2\mathrm{k}+1}$$

$$\mathrm{m} = \frac{2\mathrm{h}+1}{2\mathrm{k}} < 1$$

$$\mathrm{m} = \frac{2\mathrm{h}+1}{2\mathrm{k}+1} > 1$$

$$m = \frac{2h + 1}{2k + 1} < 1$$

The Only first type of Lame curve is superellipse, therefore, the equation for superellipse is concluded as,

$$\left(\frac{x}{a}\right)^{2/\varepsilon} + \left(\frac{y}{b}\right)^{2/\varepsilon} = 1$$

Where, ε be any +ve real number [5].

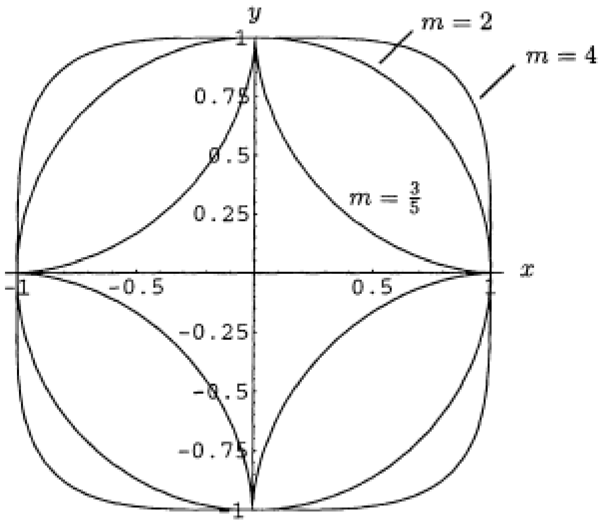

Fig. 3.1 Equation of the superellipse shows a set of continuous curves [5].

The base model of superquadrics is based on the spherical product discussed under below points.

3.1.1 Implicit and Explicit Representation of Superquadrics

The implicit equation of the basic superquadrics surface is given by,

$$|x|^p + |y|^q + |z|^r = 1$$

Where, p, q, and r are positive real numbers and that determine the various features of the superquadrics.

Such as,

For less than one: a spiked octahedron with concave faces and sharp edges.

For equal to one: a regular octahedron.

For between one and two: an octahedron with convex faces, blunt edges, and blunt corners.

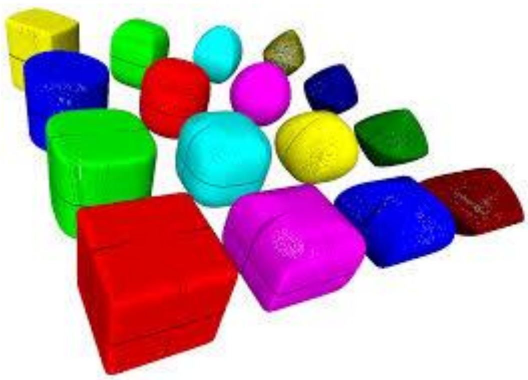

Fig. 3.2 The Family of Superellipsoid [10]

For equal to two: gives a sphere

For more than two: a cube with rounded edges and corners.

And for Infinite (in the limit): a cube

If any exponent has negative value, the shape extends to infinity. Sometimes, such shapes have the forms of super-hyperboloids [1].

3.1.2 Spherical Product

Spherical product is a trending mathematical tool that gives a 3D mathematical solid when it applied between two parametric curves [7]. By this tool, Alan H. Burr explains the mathematical analysis of Superquadrics, which is a basic explanation of Superquadrics [5].

Spherical product of two 2-D curves,

$$\underline{m}\,(\eta) = \begin{bmatrix} m_1(\eta) \\ m_2(\eta) \end{bmatrix};\ \text{where}\ \eta_0 \le \eta \le \eta_1$$

And

$$\underline{h}\,(\omega) = \begin{bmatrix} h_1(\omega) \\ h_2(\omega) \end{bmatrix};\ \text{where}\ \omega_0 \le \omega \le \omega_1$$

Gives a surface which may be denoted as $\underline{x} = \underline{m} \otimes \underline{h}$, and defined as,

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} m_1(\eta) & 0 \\ m_2(\eta) & 0 \\ 0 & 1 \end{bmatrix} X \begin{bmatrix} h_1(\omega) \\ h_2(\omega) \end{bmatrix} = \begin{bmatrix} m_1(\eta)h_1(\omega) \\ m_2(\eta)h_1(\omega) \\ h_2(\omega) \end{bmatrix};$$

$$\text{where}\ \eta_0 \le \eta \le \eta_1 \text{and}\ \omega_0 \le \omega \le \omega_1$$

Here, $\underline{m}\,(\eta)$ is a horizontal curve, that swept vertically along to the curve $\underline{h}\,(\omega)$; $h_1(\omega)$ changes the relative scale of $\underline{m}$, while $h_2(\omega)$ raises and lowers it. η is an east-west parameter, like the longitude, whereas ω is a north-south parameter like latitude. The parameter η affects the surface vertically, while ω affects the surface horizontally. The resultant surfaces have 2 degrees of freedom while the involved 2D curves have only a single degree of freedom. To rescale such a spherical product can use a separate vector $\underline{a}$ [1]. Let,

$$\underline{a} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}; then\ \underline{\widehat{m}} = \begin{bmatrix} a\ m_1(\eta) \\ b\ m_2(\eta) \end{bmatrix};\ \underline{\hat{h}} = \begin{bmatrix} h_1(\omega) \\ c\ h_2(\omega) \end{bmatrix}$$

This yielding, $\hat{x} = \underline{\widehat{m}} \otimes \underline{\hat{h}} = \begin{bmatrix} a\ m_1(\eta)h_1(\omega) \\ b\ m_2(\eta)h_1(\omega) \\ c\ h_2(\omega) \end{bmatrix}$

Geometrically, the spherical product is obtained when the half-circle,

$$\underline{m}(\eta) = \begin{bmatrix} \cos\eta \\ \sin\eta \end{bmatrix};\ where, -\pi/2 \le \eta \le \pi/2$$

is crossed out with the full circle,

$$\underline{h}(\omega) = \begin{bmatrix} \cos\omega \\ \sin\omega \end{bmatrix};\ where, -\pi \le \omega \le \pi$$

i.e. $\underline{\hat{x}} = \underline{m} \otimes \underline{h} = \begin{bmatrix} \cos\eta\cos\omega \\ \sin\eta\cos\omega \\ \sin\omega \end{bmatrix}$;

$where, -\pi/2 \le \eta \le \pi/2\ and\ -\pi \le \omega \le \pi.$

Where, the half-circle gives the radius of the southern full circle on the longitudinal axis and also drives the length above and below the X-Y plane. So, the basic trigonometric equations of the standard quadric solids and surfaces can be represented in this product form. Thus, the term "spherical product" comes from such geometrical products [1].

3.1.3 Inside - Outside Function

Let the parametric equation of the unit sphere is,

$$\underline{\hat{x}} = \underline{m} \otimes \underline{h} = \begin{bmatrix} \cos\eta\cos\omega \\ \sin\eta\cos\omega \\ \sin\omega \end{bmatrix};$$

$$where, -\pi/2 \le \eta \le \pi/2\ and\ -\pi \le \omega \le \pi$$

Then the equation of its inside–outside function is,

$$f(x,y,z) = x^2 + y^2 + z^2$$

Where, if

$$f(x,y,z) = 1, \qquad (x,y,z)\ lies\ on\ the\ surface$$

$$f(x,y,z) > 1, \qquad (x,y,z)\ lies\ outside\ the\ sphere$$

$$f(x,y,z) < 1, \qquad (x,y,z)\ lies\ inside\ the\ sphere$$

And hence an inside-outside function is a locus of a point in 3-D space relative to the given equation of surface or solid. Similarly, we have for superquadrics as discussed under the below heads.

The existence of the inside-outside functions represents that the superquadrics can be applicable for solid Boolean operations [7], e.g. union, intersection, and subtraction [1].

3.2 Types of Superquadrics (TOSq)

The family of Superquadrics mainly can be categorized in to three parts as given below,

1. Superellipsoids
2. Superhyperboloids and
3. Supertoroids

3.2.1 Superellipsoids

3.2.1.1 Ellipsoids

Equation of Ellipsoids,

General Form:

$$(^{x}/_{a})^2 + \left(^{y}/_{b}\right)^2 + (^{z}/_{c})^2 = 1$$

Trigonometric Form:

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} \cos\eta \\ c\,\sin\eta \end{bmatrix} \otimes \begin{bmatrix} a\cos\omega \\ b\,\sin\omega \end{bmatrix} = \begin{bmatrix} a\,\cos\eta\cos\omega \\ b\,\cos\eta\sin\omega \\ c\,\sin\eta \end{bmatrix}$$

$$where, -\pi/2 \le \eta \le \pi/2 \;\; and \;\; -\pi \le \omega \le \pi$$

3.2.1.2 Superellipsoids

The position vector of surface,

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} \cos^{\varepsilon_1}\eta \\ c\,\sin^{\varepsilon_1}\eta \end{bmatrix} \otimes \begin{bmatrix} a\cos^{\varepsilon_2}\omega \\ b\,\sin^{\varepsilon_2}\omega \end{bmatrix}$$

$$= \begin{bmatrix} a\,\cos^{\varepsilon_1}\eta\cos^{\varepsilon_2}\omega \\ b\,\cos^{\varepsilon_1}\eta\sin^{\varepsilon_2}\omega \\ c\,\sin^{\varepsilon_1}\eta \end{bmatrix}$$

$$\text{where}, -\pi/2 \le \eta \le \pi/2 \;\; \text{and} \;\; -\pi \le \omega < \pi$$

Equation of Normal vector,

$$\underline{n}\,(\eta, \omega,) \;\; = \begin{bmatrix} ^{1}/_{a}\,(\cos^{2-\varepsilon_1}\eta\cos^{2-\varepsilon_2}\omega) \\ ^{1}/_{b}\,(\cos^{2-\varepsilon_1}\eta\sin^{2-\varepsilon_2}\omega) \\ ^{1}/_{c}\,(\sin^{2-\varepsilon_1}\eta) \end{bmatrix}$$

Inside-outside function,

$$f(x, y, z) = \left\{(^{x}/_{a})^{2/\varepsilon_2} + \left(^{y}/_{b}\right)^{2/\varepsilon_2}\right\}^{\varepsilon_2/\varepsilon_1} + (^{z}/_{c})^{2/\varepsilon_1}$$

The family of Superellipsoids shown in Fig. 3 and here if ε_1 is the squareness parameter in the north-south direction, and ε_2 is the squareness parameter in the east-west direction then,

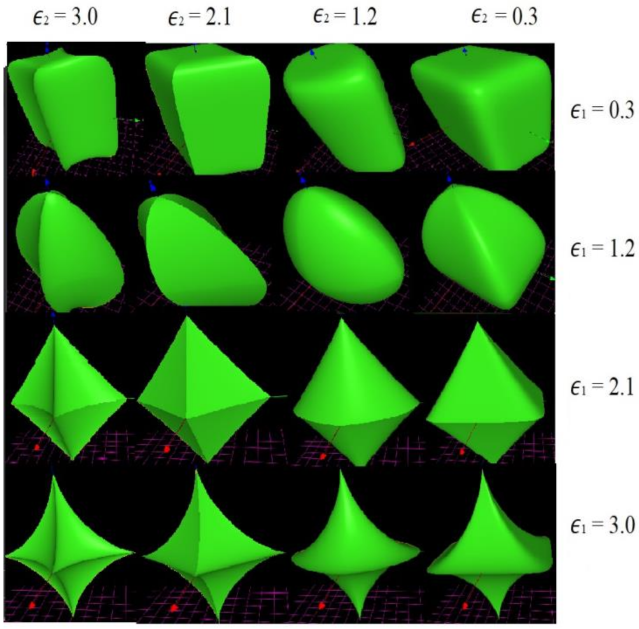

Fig. 3.3 Superquadrics: The family of Superellipsoids for exponents range from 0.3 to 3.0 achieved by using MathMod 9.1.

Cuboids are generated when both ϵ_1 and ϵ_2 are < 1.

Cylindroids are generated when ϵ_2 ~ 1 and ϵ_1 < 1.

Pillow shapes are generated when ϵ_1 ~ 1 and ϵ_2 < 1.

Flat-bevel shapes are generated when either ϵ_1 or ϵ_2 = 2.

Pinched shapes are produced when either ϵ_1 or ϵ_2 > 2.

3.2.2 Superhyperboloids

3.2.2.1 Superhyperboloids of one sheet

j) Hyperboloids of one sheet:

Equation of hyperboloid of one sheet,

General Form:

$$({}^{x}/_{a})^2 + \left({}^{y}/_{b}\right)^2 - ({}^{z}/_{c})^2 = 1$$

Trigonometric Form:

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} \sec\eta \\ c\,\tan\eta \end{bmatrix} \otimes \begin{bmatrix} a\cos\omega \\ b\,\sin\omega \end{bmatrix} = \begin{bmatrix} a\,\sec\eta\,\cos\omega \\ b\,\sec\eta\,\sin\omega \\ c\,\tan\eta \end{bmatrix}$$

$$where, -\pi/2 \le \eta < \pi/2 \;\; and \;\; -\pi \le \omega < \pi$$

k) Superhyperboloids of one-sheet

Position vector of surface,

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} \sec^{\varepsilon_1} \eta \\ c \tan^{\varepsilon_1} \eta \end{bmatrix} \otimes \begin{bmatrix} a \cos^{\varepsilon_2} \omega \\ b \sin^{\varepsilon_2} \omega \end{bmatrix}$$

$$= \begin{bmatrix} a \sec^{\varepsilon_1} \eta \cos^{\varepsilon_2} \omega \\ b \sec^{\varepsilon_1} \eta \sin^{\varepsilon_2} \omega \\ c \tan^{\varepsilon_1} \eta \end{bmatrix}$$

$$where, -\pi/2 \leq \eta < \pi/2 \;\; and \;\; -\pi \leq \omega < \pi$$

Equation of Normal Vector,

$$\underline{n}\,(\eta, \omega,) \;\; = \begin{bmatrix} 1/_a\,(\sec^{2-\varepsilon_1} \eta \cos^{2-\varepsilon_2} \omega) \\ 1/_b\,(\sec^{2-\varepsilon_1} \eta \sin^{2-\varepsilon_2} \omega) \\ 1/_c\,(\tan^{2-\varepsilon_1} \eta) \end{bmatrix}$$

Inside-outside function,

$$f(x, y, z) = \left\{ (x/a)^{2/\varepsilon_2} + \left(y/b\right)^{2/\varepsilon_2} \right\}^{\varepsilon_2/\varepsilon_1} - (z/c)^{2/\varepsilon_1}$$

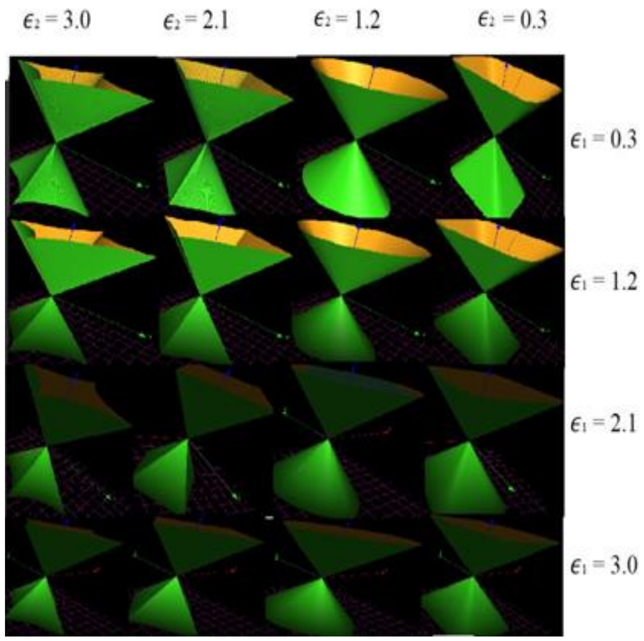

Fig. 3.4 Superquadrics: The Family of Superhyperboloids of one piece for exponents range from 0.3 to 3.0 achieved by using MathMod 9.1.

3.2.2.2 Superhyperboloids of two sheet

Hyperboloids of two sheet:

Equation of Hyperboloids of two-sheet

General Form:

$$(x/a)^2 - \left(y/b\right)^2 - (z/c)^2 = 1$$

Trigonometric Form:

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} \sec\eta \\ c\tan\eta \end{bmatrix} \otimes \begin{bmatrix} a\sec\omega \\ b\tan\omega \end{bmatrix} = \begin{bmatrix} a\sec\eta\sec\omega \\ b\sec\eta\tan\omega \\ c\tan\eta \end{bmatrix}$$

$$where, -\pi/2 < \eta < \pi/2\,;$$

$$-\pi/2 < \omega < \pi/2 \;\; (For\ Piece\ 1),$$

$$and\ \pi/2 < \omega < 3\pi/2 \;\; (For\ Piece\ 2)$$

Superhyperboloids of two-sheet

The Position vector of surfaces:

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} \sec^{\varepsilon_1}\eta \\ c\tan^{\varepsilon_1}\eta \end{bmatrix} \otimes \begin{bmatrix} a\sec^{\varepsilon_2}\omega \\ b\tan^{\varepsilon_2}\omega \end{bmatrix}$$

$$= \begin{bmatrix} a\sec^{\varepsilon_1}\eta\sec^{\varepsilon_2}\omega \\ b\sec^{\varepsilon_1}\eta\tan^{\varepsilon_2}\omega \\ c\tan^{\varepsilon_1}\eta \end{bmatrix}$$

$$where, -\pi/2 < \eta < \pi/2\,;$$

$$-\pi/2 < \omega < \pi/2 \;\; (For\ Piece\ 1),$$

$$and\ \pi/2 < \omega < 3\pi/2 \;\; (For\ Piece\ 2)$$

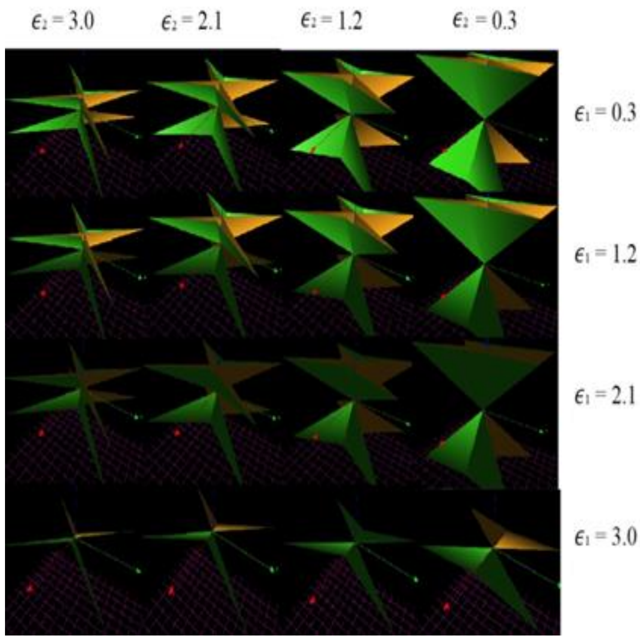

Fig. 3.5 Superquadrics: Family of Superhyperboloids of two-piece for exponents range from 0.3 to 3.0 achieved by using MathMod 9.1.

Equation of Normal Vector:

$$\underline{n}\,(\eta, \omega,) \;= \begin{bmatrix} {}^{1}\!/_{a}\,(\sec^{2-\varepsilon_1}\eta \sec^{2-\varepsilon_2}\omega) \\ {}^{1}\!/_{b}\,(\sec^{2-\varepsilon_1}\eta \tan^{2-\varepsilon_2}\omega) \\ {}^{1}\!/_{c}\,(\tan^{2-\varepsilon_1}\eta) \end{bmatrix}$$

Inside-outside function:

$$f(x,y,z) = \left\{ ({}^{\mathrm{x}}\!/_{\mathrm{a}})^{2/\varepsilon_2} - \left({}^{\mathrm{y}}\!/_{\mathrm{b}}\right)^{2/\varepsilon_2} \right\}^{\varepsilon_2/\varepsilon_1} - ({}^{\mathrm{z}}\!/_{\mathrm{c}})^{2/\varepsilon_1}$$

3.2.1 Supertoroids

A torus is an extended form of a quadric surface.

Position vector of surface:

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} d + \cos^{\varepsilon_1}\eta \\ c\,\sin^{\varepsilon_1}\eta \end{bmatrix} \otimes \begin{bmatrix} a\cos^{\varepsilon_2}\omega \\ b\,\sin^{\varepsilon_2}\omega \end{bmatrix}$$

$$= \begin{bmatrix} a\,(d + \cos^{\varepsilon_1}\eta)\cos^{\varepsilon_2}\omega \\ b\,(\mathrm{d} + \cos^{\varepsilon_1}\eta)\sin^{\varepsilon_2}\omega \\ c\,\sin^{\varepsilon_1}\eta \end{bmatrix} where, -\pi \le \eta < \pi \; and \; -\pi \le \omega < \pi$$

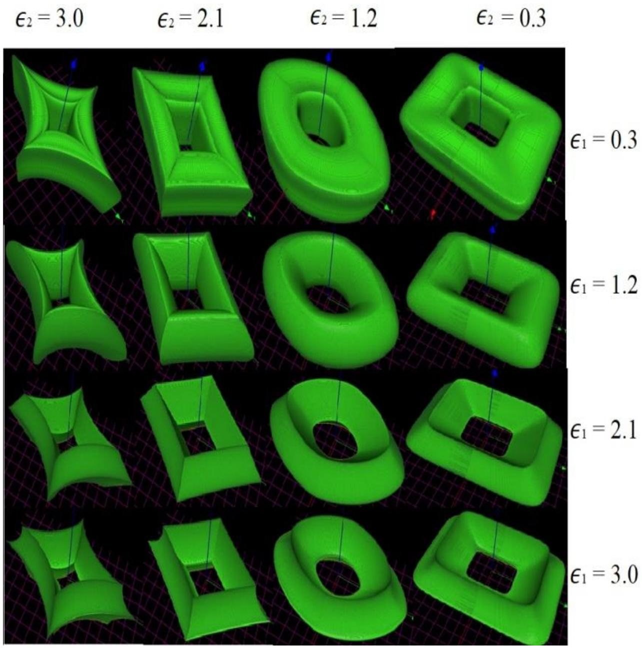

Fig. 3.6 Superquadrics: Family of Supertoroids for exponents range from 0.3 to 3.0 achieved by using MathMod 9.1.

Equation of Normal Vector:

$$\underline{n}\,(\eta,\omega,) = \begin{bmatrix} 1/_a\,(\cos^{2-\varepsilon_1}\eta\cos^{2-\varepsilon_2}\omega) \\ 1/_b\,(\cos^{2-\varepsilon_1}\eta\sin^{2-\varepsilon_2}\omega) \\ 1/_c\,(\sin^{2-\varepsilon_1}\eta) \end{bmatrix}$$

Inside-outside function:

$$f(x,y,z) = \left\{\left((^x/_a)^{2/\varepsilon_2} + \left(^y/_b\right)^{2/\varepsilon_2}\right)^{\varepsilon_2/2} - d\right\}^{2/\varepsilon_1} + (^z/_c)^{2/\varepsilon_1}$$

Where, $d = \hat{r}/\sqrt{(a^2 + b^2)}$, and where $\hat{r}$ is the radius of torus that revolves along with the circumference of another circle of radius R. Here,

The inside function of solid is given by f (x, y, z) < 1.

The surface is given by f (x, y, z) = 1.

And the outside function is given by f (x, y, z) > 1.

3.3 Canonical Position of Superquadrics

There is a group of four parametric curves which is the base of superquadrics solids in which the trigonometric terms raised to exponents [5]. To understand let be consider for 2-D first, the below given sin-cosine curve forms the parametric equation of superellipse,

$$x = a\cos^{\varepsilon}\theta\,, y = b\cos^{\varepsilon}\theta\,, where, -\pi \le \theta < \pi$$

and the general form is,

$$\left(\frac{x}{a}\right)^{2/\varepsilon} + \left(\frac{y}{b}\right)^{2/\varepsilon} = 1$$

While the secant-tangent curve as given below is called superhyperbola,

$$x = a\sec^{\varepsilon}\theta_1\,, y = b\tan^{\varepsilon}\theta_2,$$

$$\text{where } -{}^{\pi}/_2 \le \theta_1 < {}^{\pi}/_2 \text{ and } {}^{\pi}/_2 \le \theta_2 \le {}^{3\pi}/_2$$

The general form is, $\left(\frac{x}{a}\right)^{2/\varepsilon} - \left(\frac{y}{b}\right)^{2/\varepsilon} = 1$

Now, the spherical product of couple of such types of equations gives a uniform 3-D solid [8]. In this way, we get two exponents known as squareness parameters in north-south and east-west direction respectively or it can be taken as per the global coordinate system. But the point, which needs to say here is both exponents defines the shape of solids as below,

$\varepsilon < 1$: gives square shape; $\varepsilon > 2$: gives pinched shape;

$\varepsilon \sim 1$: gives round shape; $\varepsilon \sim 2$: it gives a flat bevel

CHAPTER 4: MODELING EXPERIMENTS AND FINDINGS

4.1 POLY MODELS OF SUPERQUADRICS (PMSq)

4.1.1 Gridded Superquadrics (GSq):

In Design & Modeling, A grid is a unit divisional part of 2-D image or 3-D model, divided, based on a limit described on a specific criterion as per the analytical requirements of such a design or model. And, any 2-D Design or 3-D Model can be divided into a maximum definite number of partitions by polylines and curves, by which those are formed in geometry. And, therefore polylines and curves refer to those sequential lines and curves which are used in the formation of individual 2-D design or structure of 3-D model and they are the lowest existing part of the design or model [3].

When we take its reverse and make a model or object using computer graphics and mathematical formulation, we getting various shape and size of a single mathematical equation with respect to various grid sizes. Here we applied the same on superquadrics and have an extended family of superquadrics called Gridded Superquadrics (GSq) here in this paper.

4.1.1.1 Core Concept:

In mathematical modeling, the size of grids and numbers of polylines affect the shape of the objects [3]. To make a better understanding about this concept, let us consider an example.

As we know that the parametric equation of the unit sphere is,

$$\underline{\hat{x}} = \underline{m} \otimes \underline{h} = \begin{bmatrix} \cos\eta \cos\omega \\ \sin\eta \cos\omega \\ \sin\omega \end{bmatrix};$$

$$where, -\pi/2 \leq \eta \leq \pi/2 \;\; and \;\; -\pi \leq \omega \leq \pi$$

The geometrical modeling of this mathematical equation of the sphere is shown in figs 3.1.1 (1 to 6).

Here, in the first fig 3.1.1 (1), shows the sphere for the very low size of grid i.e. 4 X 4 and no. of polylines are 8. As, this model looks like a polygonal and but the mathematical equation is of the sphere when we increase the grid size and no. of poly line getting the formation of spherical shape as this formation is presented in further figs 3.1.1(2 to 5) and the last fig 3.1.1 (5) which gives the geometrical model of sphere on the grid size 2400 X 2400 and 2878800 is the no. of polylines, which is a higher value of grid size and polylines also. And, this model gives a good shape of a sphere with good surface quality. As, the better view of surface quality is presented in fig. 3.1.1 (6) without the show of polylines, also called mesh view.

Similarly, a single mathematical equation of a subfamily of superquadrics presents a hybrid family of various solids and surfaces for a single set of exponent values.

Furthermore, those models of superquadrics represent a unique set and therefore they may useful in robotics [7] and in the designing of video games.

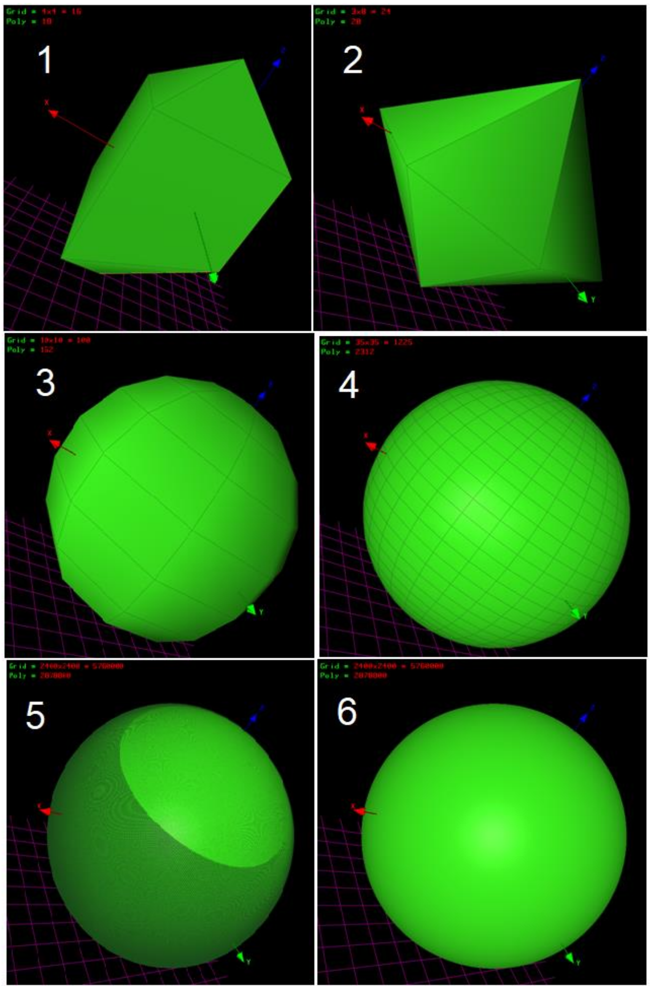

Fig. 4.1 Mathematical models of the Sphere

For,

1. Very low quantity of Grid = 4 X 4 and Poly = 18
2. Low quantity of Grid = 3 X 8, and Poly = 28
3. Medium quantity of Grid = 10 X 10, and Poly = 162
4. High quantity of Grid = 35 X 35, and Poly = 2312
5. Very high quantity of Grid = 2400 X 2400, and Poly = 2878800
6. Grid = 2400 X 2400, and Poly = 2878800, Mesh view

4.1.1.2 Modeling of Superquadrics with Grid Size (Sq-WGS):

Various Grid size makes various models for a set of parameters of superquadrics equation. Those models are different in their morphology, surface quality, and in the view of other geometrical aspects also.

Here in this section we presenting for some grid size as an example and try to make an understanding about the aspects and kind of modeling.

4.1.1.3 Superellipsoids with Grid Size (Se-WGS):

The position vector of surface,

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} \cos^{\varepsilon_1}\eta \\ c\,\sin^{\varepsilon_1}\eta \end{bmatrix} \otimes \begin{bmatrix} a\cos^{\varepsilon_2}\omega \\ b\,\sin^{\varepsilon_2}\omega \end{bmatrix}$$

$$= \begin{bmatrix} a\cos^{\varepsilon_1}\eta\cos^{\varepsilon_2}\omega \\ b\cos^{\varepsilon_1}\eta\sin^{\varepsilon_2}\omega \\ c\sin^{\varepsilon_1}\eta \end{bmatrix}$$

$$\text{where,} -\pi/2 \le \eta \le \pi/2 \text{ and } -\pi \le \omega < \pi$$

$$\le \omega \le \pi$$

When modeling this equation of Superellipsoid using MathMod 9.1 software for size parameter, a = 10, b = 20, c = 30, exponent value combination, $\varepsilon_1 = 2.5$, $\varepsilon_2 = 0.1$,under the angular limit $-\pi/2 \le \eta \le \pi/2$ and $-\pi \le \omega < \pi$. We get figs 3.2.1 (1-4), with grid size 3 X 3, 84 X 73, 73 X 73, 107 X 92 and Poly = 8, 11952, 10368, 19292 respectively.

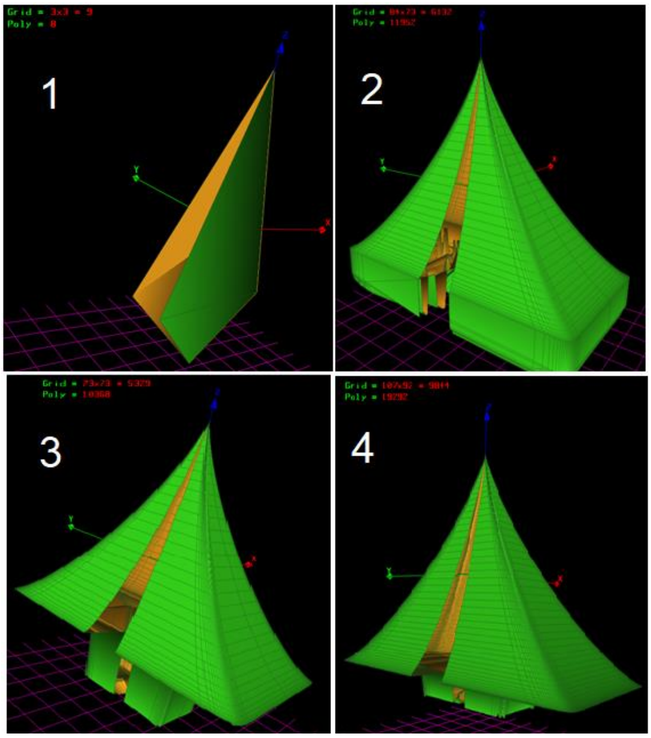

Fig. 4.2 Superellipsoids for size parameter a = 10, b = 20, c = 30, exponent value, $\varepsilon_1 = 2.5, \varepsilon_2 = 0.1$, under limit $-\pi/2 \le \eta \le \pi/2$ and $-\pi \le \omega < \pi$ and

1. For Grid Size 3 X 3 and Poly = 8,
2. For Grid Size 84 X 73 and Poly = 11952,
3. For Grid Size 73 X 73 and Poly = 10368,
4. For Grid Size 107 X 92 and Poly = 19292,

These models shows pinched shape with large area base, open section are due to limit error, which can remove or closed the section by increasing the limit.

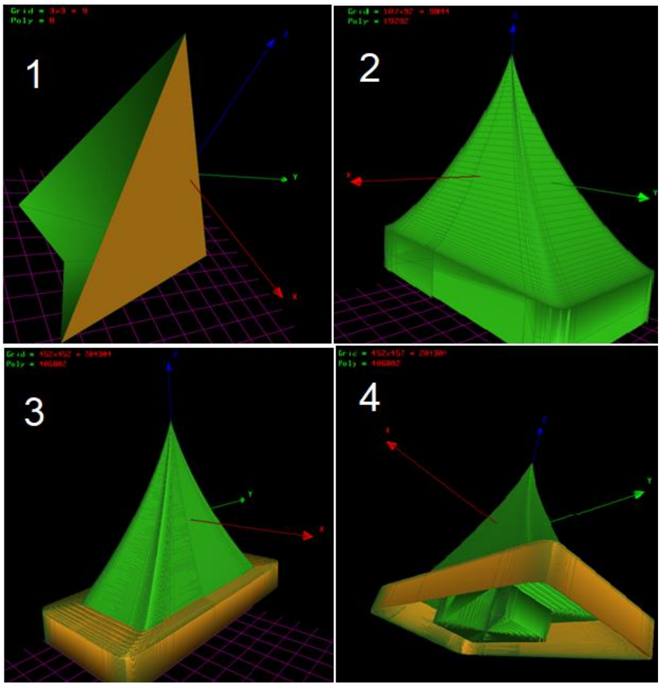

Fig. 4.3 Superellipsoids for size parameter a = 10, b = 20, c = 30, exponent value, $\varepsilon_1 = 2.5, \varepsilon_2 = 0.1$, under limit $-\pi/2 \leq \eta \leq \pi/2$ and $-\pi \leq \omega < 2 * \pi$ and

1. For Grid Size 3 X 3 and Poly = 8,
2. For Grid Size 107 X 92 and Poly = 19292,
3. & 4) For Grid Size 452 X 452 and Poly = 406802,

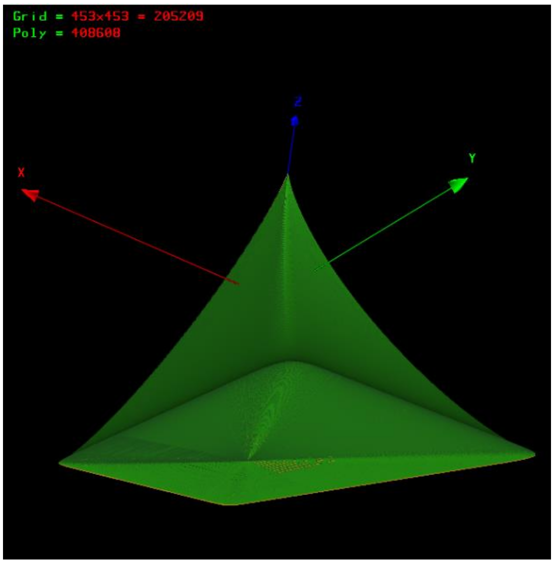

Fig. 4.4 Superellipsoids for size parameter a = 10, b = 20, c = 30, exponent value, $\varepsilon_1 = 2.5, \varepsilon_2 = 0.1$, under limit $-\pi/2 \leq \eta \leq \pi/2$ and $-\pi \leq \omega < 2 * \pi$ and For Grid Size 453 X 453 and Poly = 408608

That closed model found by increased limit as from $-\pi/2 \leq \omega < \pi/2$ to $-\pi \leq \omega < 2 * \pi$ as such models are shown in figs 3.2.2 (1-4) with grid size 3 X 3, 107 X

92, 452 X 452, 452 X 452 , and Poly = 8, 19292, 406802, 406802 respectively and fig. 3.2.3 for grid size = 453 X 453 and poly = 408608.

Where these models show a pinched corner [5] at one end, there they show a rectangular shape at another opposite end (see fig 3.2.2 (4)). This model reflects with rectangular open like bucket section at its pinched corner and rectangular protuberant shape at its opposite end (see fig 3.2.3) [5].

In the similar manner fig 3.2.4 (1-6) shows superellipsoid for size parameter, a = 10, b = 20, c = 30, exponent value combination, $\varepsilon_1 = 2.5$, $\varepsilon_2 = 1.0$,

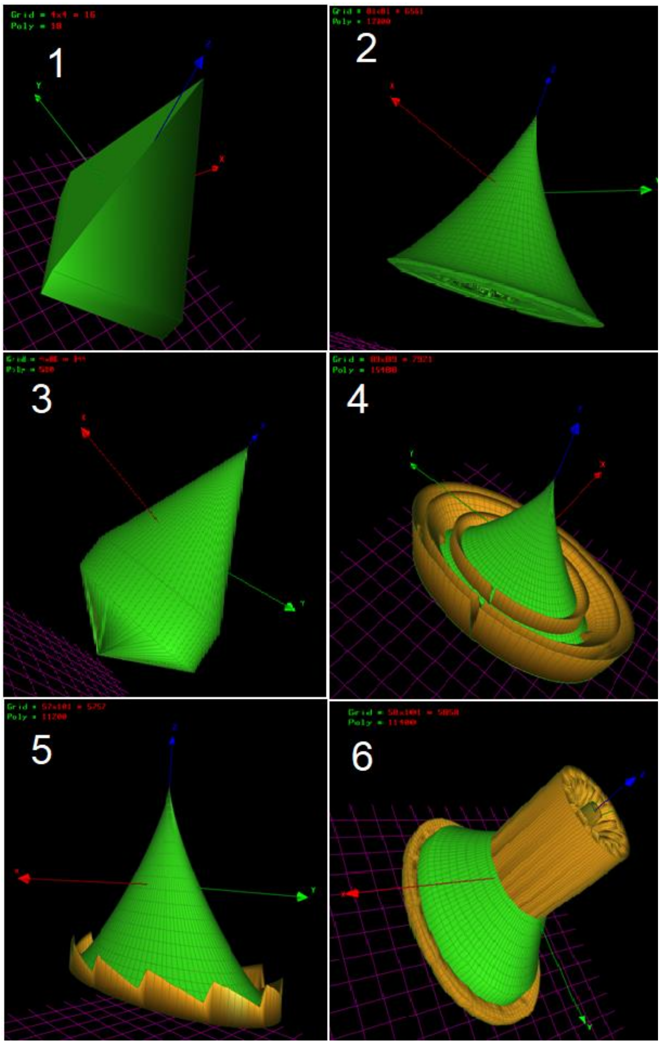

Fig. 4.5 Superellipsoids for size parameter a = 10, b = 20, c = 30, exponent value, $\varepsilon_1 = 2.5, \varepsilon_2 = 1.0$, under limit $-\pi/2 \leq \eta \leq \pi/2$ and $-\pi \leq \omega < \pi$ and

1. For Grid Size 4 X 4 and Poly = 18,
2. For Grid Size 107 81 X 81 and Poly = 12800,
3. For Grid Size 4 X 86 and Poly =510,
4. For Grid Size 89 X 89 and Poly = 15488,
5. For Grid Size 57 X 101 and Poly = 11200,
6. For Grid Size 58 X 101 and Poly = 11400,

under the angular limit $-\pi/2 \leq \eta \leq \pi/2$ and $-\pi \leq \omega < \pi$ for grid Sizes 4 X 4, 81 X 81, 4 X 86, 89 X 89, 57 X 101, 58 X 101 and poly no. = 18, 12800, 510, 15488, 11200, 9800 respectively.

These models show a family of large no. of the various model varies with concerning grid density and poly no. and the same thing is seen for other remaining valid combinations of exponent values. And similarly, for the remaining other superquadrics such as superhyperboloids of one-sheet, two-sheet, and supertoroids family, we find the multiple shapes for a single combination of exponents.

This appearance of such poly models shows a large extension of these superquadrics family by using various grid sizes, poly no. and limits. And, those extended models can be used like as other models of superquadrics at many places as per the requirements.

4.2 TRI EXPONENTIAL SUPERQUADRICS (TESq)

4.2.1 Background

The set of variables play an important role in the geometrical modeling of mathematical equations. These variables are responsible for the morphology of the model. In the case of Superquadrics, they give a lot of Various correlated models with respect to their different combinations of numerical values and limits; and therefore the superquadrics represent a hybrid family of solids and surfaces. If move a bit further, as a result it has categorized mainly in three categories, Superellepsoids, Superhyperboloids, and Supertoroids [1].

After reviewing of qualitative and analytical modeling presentation of D. Paschalidou et al [11], it is clear that, the design, modeling, and reconstruction of complex 3D shapes can be made easily by using superquadrics [9] and its primitives. As they directed to the inclusion of parameters for extension of such models, here we present a modeling test and its observations using the third exponent in the mathematical equations of superquadrics.

4.2.2 Conceptual Details

Over the past decade, A distinguished Interest in the study of the expansion of these superquadrics have been seen in the researchers. Along with this, Research on various complex designs, sampling, primitive formation, and modeling of complex object and machinery have been seen. Here before proceeding further we refer to the mathematical preliminaries explained by A. H. Barr in his seminal paper of 1981 [1]. In the mathematical modeling of those superquadrics equations we see a hybrid family of 3D solids and surfaces. Such models are due to shape parameters which are also known as squareness parameters and used in terms of two exponents. And, these two exponents give different shape changes in both 2D direction respectively as discussed there [1]. Further, here we taking another exponent, that affects the shape in the third direction or third axis in the 3-D coordinate system. Similar to that we taking a spherical product of two curves and getting position vector of respective superquadrics family with the third exponent [5], as shown in the equations of position vectors in the next section 2 respectively for all three families of superquadrics. And further, we present the geometrical modeling [6] of such position vectors by using the software 'MathMod 9.1' as shown in figs 2.1.1, 2.2.1, 2.2.2, and 2.3.1 for superellipsoids. Superhuyperboloids of one sheet and two-sheet, and for supertoroids respectively. Here also presenting the mathematical equation of normals of such families in their respective subsections.

It is great to us that we have existing geometrical models for these new mathematical equations and here presenting for the range from 1 to 4 of the second and the third exponent value combination by putting the first exponent at the unit. That is a valid proof of existing of these geometrical models [6] and mathematical representation of superquadrics families with the third exponent.

Thus, it is the presentation of the expended family of superquadrics and that models can be used in many applications like other pre members of superquadrics family.

Here also we have presented another classification with proper reason and valid specifications of these Superquadrics with a third exponent (SqWTE), called as Even Suerquadrics (ESq) and Odd Superquadrics (OSq).

4.2.3 Modeling of Superquadrics with Third Exponent (SqWTE)

Various kind of mathematical and geometrical modeling of superellipsoids with proofs was presented by many authors [1, 2] and we have good discussions about its application and advancement in the field of design and modeling of complex objects.

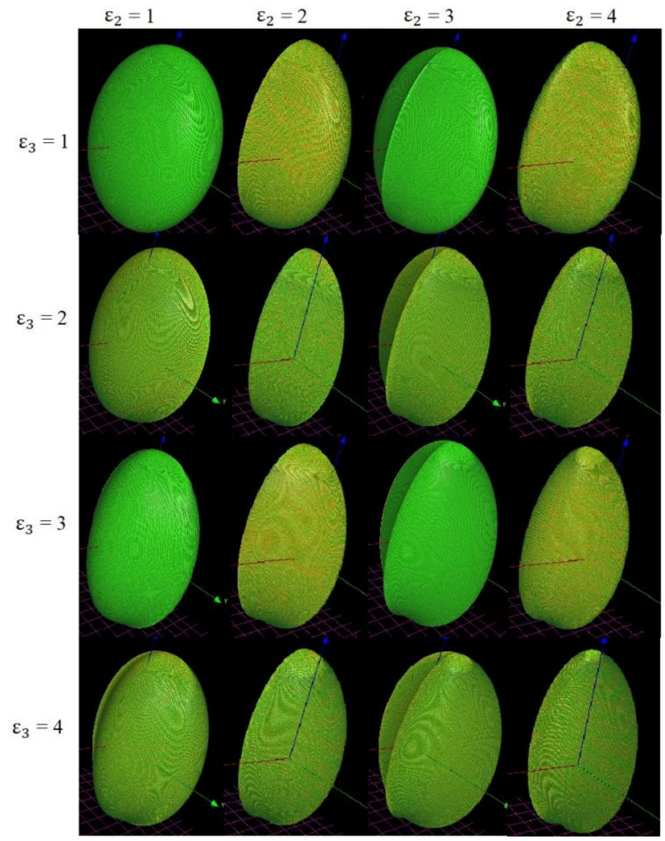

Fig. 4.6 Set of Models of Superellipsoids with the third exponent (ε_3), taking$\varepsilon_1 =$ 1.0, Made in Math Mod 9.1, on the grid size 500 X 500.

In such a way, here we have to present an additional extension to the family of superellipsoids by introducing its third exponent. Both the mathematical and geometrical modeling is discussed here with few specific, valid, and important models of its extended family.

4.2.3.1 Superellipsoids with third exponent (Se-WTE)

The position vector of the surface of superellipsoids with the consideration of the third exponent is given as below the spherical product of two distinct curves,

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} \cos^{\varepsilon_1}\eta \\ c\,\sin^{\varepsilon_1}\eta \end{bmatrix} \otimes \begin{bmatrix} a\cos^{\varepsilon_2}\omega \\ b\,\sin^{\varepsilon_3}\omega \end{bmatrix}$$

$$= \begin{bmatrix} a\cos^{\varepsilon_1}\eta\cos^{\varepsilon_2}\omega \\ b\cos^{\varepsilon_1}\eta\sin^{\varepsilon_3}\omega \\ c\sin^{\varepsilon_1}\eta \end{bmatrix}$$

$$\text{where}, -\pi/2 \le \eta \le \pi/2 \text{ and } -\pi \le \omega < \pi$$

And, its geometrical modeling is shown in below fig. 2.1.1 by taking $\varepsilon_1 = 1$ as constant and combination of remaining two exponent values under the range from 1 to 4.

And the equation of Normal vector of these surface is given as below,

$$\underline{n}\,(\eta, \omega,) = \begin{bmatrix} 1/a\,(\cos^{2-\varepsilon_1}\eta\cos^{2-\varepsilon_2}\omega) \\ 1/b\,(\cos^{2-\varepsilon_1}\eta\sin^{2-\varepsilon_3}\omega) \\ 1/c\,(\sin^{2-\varepsilon_1}\eta) \end{bmatrix}$$

Here, $\varepsilon_1, \varepsilon_2$, and ε_3 are the squareness parameters w.r.t. 3-dimensions respectively.

As we can see figure 2.1.1, the equation of superellipsoid with the third exponent represents a family of solids and here we notice that shape of models for even values of the second exponent are similar up to a limit and similarly for odd value too, but they are different with each other.

In case of even values of the second exponent models show half and open shape like a bucket of ellipsoid shape while for odd value it shows closed shape of ellipsoids and if taking value more than one for the second exponent then getting a narrow shrink lining as shown for $[\varepsilon_2, \varepsilon_3] = [3,1]; [3,2]; [3,3]$and $[3,4]$ in the above figure 2.1.1.

But all members of this family as shown in fig. 2.1.1 having some differences among them, therefore it can be claimed as a family of unique shapes and solids. And which represents an extension of the superellipsoids family by considering the third exponent.

4.2.3.2 Superhyperboloids with third exponent (SH-WTE):

l) Superhyperboloids of one sheet with third exponent (Sh1-WTE)

Like as the previous section of Superellipsoids, here the position vector of surface of superhyperboloids of one sheet with consideration of third exponent, is given as below following the spherical product of two distinct curves,

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} \sec^{\varepsilon_1} \eta \\ c \tan^{\varepsilon_1} \eta \end{bmatrix} \otimes \begin{bmatrix} a \cos^{\varepsilon_2} \omega \\ b \sin^{\varepsilon_3} \omega \end{bmatrix}$$

$$= \begin{bmatrix} a \sec^{\varepsilon_1} \eta \cos^{\varepsilon_2} \omega \\ b \sec^{\varepsilon_1} \eta \sin^{\varepsilon_3} \omega \\ c \tan^{\varepsilon_1} \eta \end{bmatrix}$$

$$where, -\pi/2 \le \eta < \pi/2 \;\; and \;\; -\pi \le \omega < \pi$$

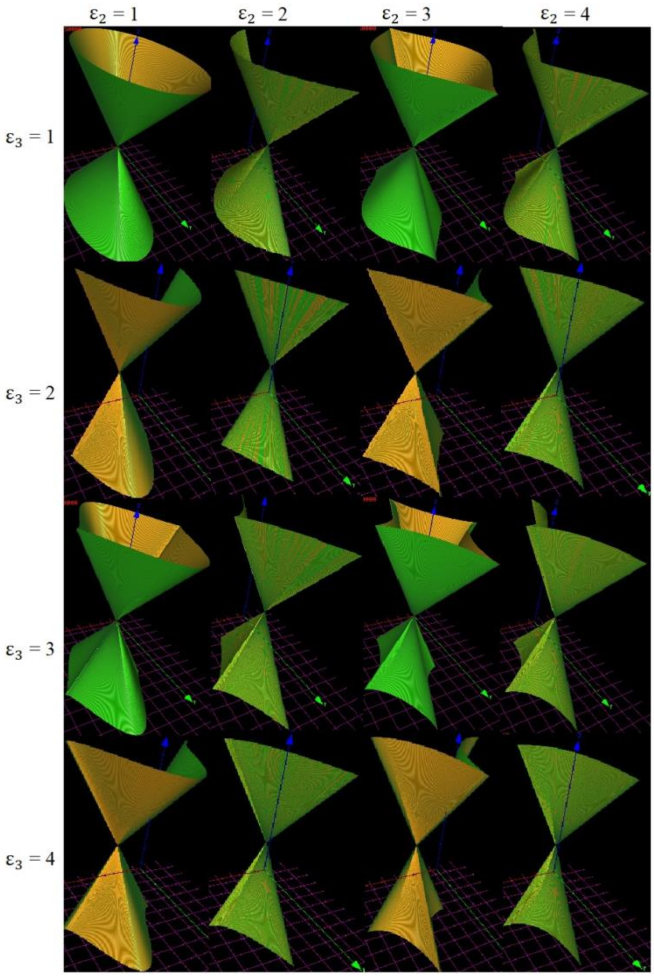

Fig. 4.7 Set of Models of Superhyperboloids of one sheet with third exponent (ε_3), taking $\varepsilon_1 = 1.0$ constant, Made in MathMod 9.1, on the grid size 500 X 500.

And, the geometrical models for various combinations of exponent values are shown in fig. 2.2.1.1 by taking $\varepsilon_1 = 1$ as constant and combination of remaining two exponent values under the range from 1 to 4.

Similar to the case superellipsoids, here too we have closed models for odd values of the second exponent and open models for even values as we can also see in figure. The mathematical equation of normal vectors for these surfaces is given as below,

Equation of Normal Vector

$$\underline{n}\,(\eta, \omega,) \;\; = \begin{bmatrix} 1/_a\,(\sec^{2-\varepsilon_1}\eta\cos^{2-\varepsilon_2}\omega) \\ 1/_b\,(\sec^{2-\varepsilon_1}\eta\sin^{2-\varepsilon_3}\omega) \\ 1/_c\,(\tan^{2-\varepsilon_1}\eta) \end{bmatrix}$$

2) Superhyperboloids of two-sheets with third exponent (Sh2-WTE)

The position vector of surfaces of superhyperboloids of two sheets with considering third exponent may be written as below, following the spherical product of these two distinct curves,

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} \sec^{\varepsilon_1}\eta \\ c\tan^{\varepsilon_1}\eta \end{bmatrix} \otimes \begin{bmatrix} a\sec^{\varepsilon_2}\omega \\ b\tan^{\varepsilon_3}\omega \end{bmatrix}$$

$$= \begin{bmatrix} a\sec^{\varepsilon_1}\eta\sec^{\varepsilon_2}\omega \\ b\sec^{\varepsilon_1}\eta\tan^{\varepsilon_3}\omega \\ c\tan^{\varepsilon_1}\eta \end{bmatrix}$$

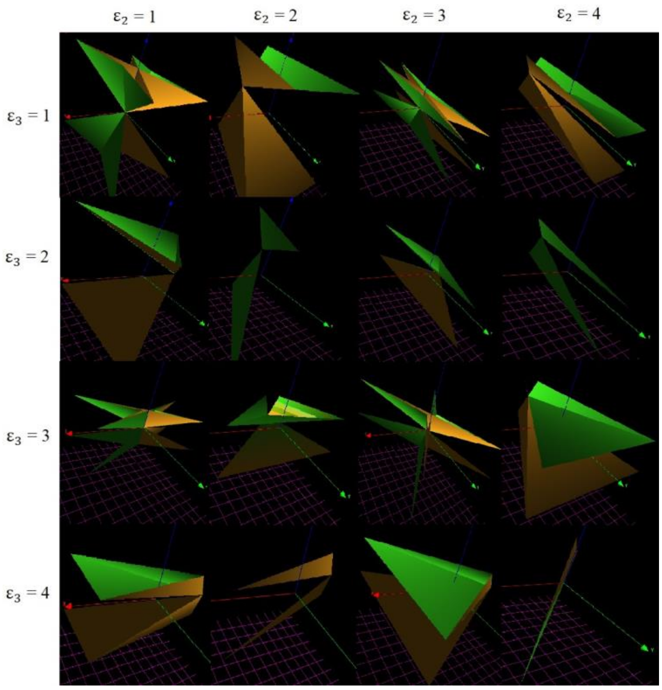

Fig. 4.8 Superhyperboloids of two sheet with third exponent (ε_3), taking $\varepsilon_1 = 1.0$ constant, Made in MathMod 9.1, on the grid size 7 X 7.

$$where, -\pi/2 < \eta < \pi/2\,;$$

$$-\pi/2 < \omega < \pi/2 \;\; (For\ Piece\ 1),$$

$$and\ \pi/2 < \omega < 3\pi/2 \;\; (For\ Piece\ 2)$$

It's geometrical modeling for the said range and combination of values as in previous sections are presented in fig. 2.2.2.1. Here during the geometrical modeling of these superellipsoids of two-sheets with the third exponent, we encountered an error. We have not found a proper model for said grid size 500 X 500, while on the lower grid size i.e. 7 X 7, we have found these presented models in figure for remaining same configuration such as range and combinations.

Normal vectors for these surfaces may be written as previous sections, and that is given as below.

Equation of Normal Vector:

$$\underline{n}\,(\eta, \omega,) \;= \begin{bmatrix} {}^{1}\!/_{a}\,(\sec^{2-\varepsilon_1}\eta\,\sec^{2-\varepsilon_2}\omega) \\ {}^{1}\!/_{b}\,(\sec^{2-\varepsilon_1}\eta\,\tan^{2-\varepsilon_3}\omega) \\ {}^{1}\!/_{c}\,(\tan^{2-\varepsilon_1}\eta) \end{bmatrix}$$

4.2.3.3 Supertoroids with third exponent (St-WTE)

The position vector of surfaces of supertoroids with third exponent is given as below,

$$\underline{x}\,(\eta, \omega,) = \begin{bmatrix} d + \cos^{\varepsilon_1}\eta \\ c\,\sin^{\varepsilon_1}\eta \end{bmatrix} \otimes \begin{bmatrix} a\cos^{\varepsilon_2}\omega \\ b\,\sin^{\varepsilon_3}\omega \end{bmatrix}$$

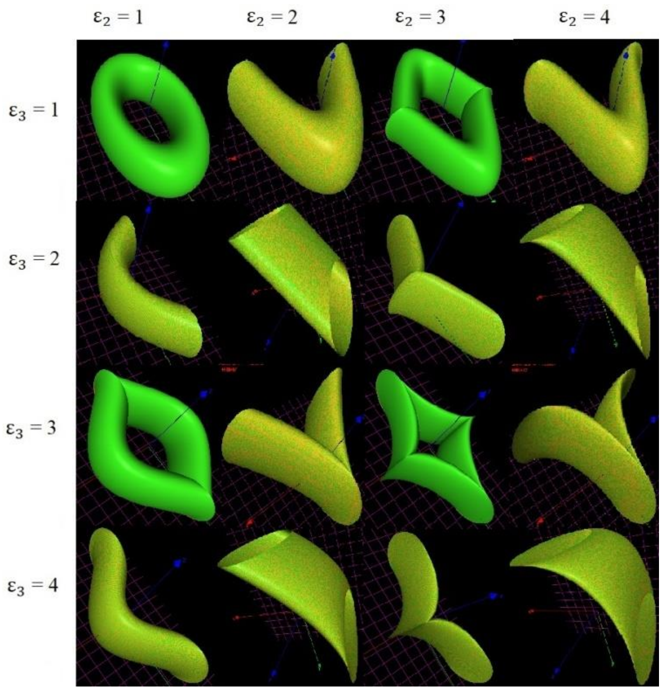

Fig. 4.9 Supertoroids with third exponent (ε_3), taking $\varepsilon_1 = 1.0$ constant, Made in MathMod 9.1, on the grid size 500 X 500.

$$= \begin{bmatrix} a\,(d + \cos^{\varepsilon_1}\eta)\cos^{\varepsilon_2}\omega \\ b\,(\mathrm{d} + \cos^{\varepsilon_1}\eta)\sin^{\varepsilon_3}\omega \\ c\,\sin^{\varepsilon_1}\eta \end{bmatrix}$$

$$where, -\pi \le \eta < \pi \; and \; -\pi \le \omega < \pi$$

And, the geometrical modeling of this equation of position vectors presented in fig 2.3.1 on the grid size 500 X 500 and where we have unique models for each combination of exponents' value. As, in figure shown for $\varepsilon_1 = 1$ taking as constant here and combination of remaining two exponents' values under the range from 1 to 4.

As in figure, some similarities can be seen clearly for even values of the second exponent but it is the different case from other previous SqWTE i.e. superellipsoids and superhyperboloids. Because, here in this figure only four models are found with closed surface, as they are presented by the combination of $[\varepsilon_2, \varepsilon_3] = [1,1]; [3,1]; [1,3]$and $[3,3]$ and remaining combinations gives open models. In that four models except $[\varepsilon_2, \varepsilon_3] = [1, 1]$, we see 2 bending line s / section at the equal sectional interval and when increasing the value getting up to four sharp bending corners as shown for the combination of $[\varepsilon_2, \varepsilon_3] = [3,3]$. If we make a look on the behavior of open sections or say likely to even models, for smaller value combinations equation shows curved, bent tube-like models as can be seen for the combination of $[\varepsilon_2, \varepsilon_3] = [2,1]; [1, 2]; [4, 1];$ and $[1, 4];$ but in the increasing order of exponent value, the shape going to be sharp and flat bent section as shown for the exponent' value combination of $[\varepsilon_2, \varepsilon_3] = [2, 3]; [3, 2]; [4,3];$ and $[3, 4];$ in figure 2.3.1.

Mathematical equation for normal vectors for these surfaces of supertoroids with third exponent is given as below,

Equation of Normal Vector:

$$\underline{n}\,(\eta, \omega,) \;=\; \begin{bmatrix} 1/_{a}\,(\cos^{2-\varepsilon_1}\eta\,\cos^{2-\varepsilon_2}\omega) \\ 1/_{b}\,(\cos^{2-\varepsilon_1}\eta\,\sin^{2-\varepsilon_3}\omega) \\ 1/_{c}\,(\sin^{2-\varepsilon_1}\eta) \end{bmatrix}$$

Where, $\mathrm{d} = \hat{\mathrm{r}}/\sqrt{(\mathrm{a}^2 + \mathrm{b}^2)}$, and where $\hat{\mathrm{r}}$ is the radius of torus that revolves along with the circumference of another circle of radius R.

4.2.4 Even and Odd Superquadrics:

In the case of Superellipsoids, all models for even value of the second exponent i.e. $\varepsilon_2 = 2, 4$ as presented in fig. 2.1.1, are even models as they all have specific similarities that is open section and remaining models are odd which are for odd value of the second exponent and they all have closed section superellipsoids. And, in which specifically four odd combinations which are $[\varepsilon_2, \varepsilon_3] = [1,1]; [3,1];$ $[1,3]$and $[3,3]$, presented highly odd models and they can be clearly differentiated among the other Superellipsoids with third exponent (SqWTE) as shown in fig. 2.1.1, and they are called as Odd Superellipsoids (OSe) and remaining combination shows even superellipsoids (ESq).

But in the case of Superhyperboloids of both types, we have found even models for even values of both the second and the third exponent, which have an open section. In the case of one-sheet superhyperboloids as shown in fig. 2.2.1.1 for the combinations of $[\varepsilon_2, \varepsilon_3] = [2,1]; [2, 2]; [2, 3]; [2,4];$

$[4,1]$; $[4,2]$; $[4,3]$; $[4\ 4]$; $[1,2]$; $[1,4]$; $[3,2]$ and $[3,4]$; are said even superhyperboloids (ESH) and remaining models of the four odd value combinations $[\varepsilon_2, \varepsilon_3] = [1,1]$; $[3,1]$; $[1,3]$;

and $[3,3]$; are called odd superhyperboloids (OSq), which have closed section. This same behavior is shown by the models of the second type of the superhyperboloids i.e. two sheeted superhyperboloids as shown in fig. 2.2.2.1 and also by the supertoroids, that's presented in fig. 2.3.1 where we have four odd supertoroids (OSt) and twelve even supertoroids (ESt).

Thus if we summarize for all these three kinds of SqWTE then we have four odd models in each type of SqWTE for the odd combination of the second and the third exponent i.e. $[\varepsilon_2, \varepsilon_3] = [1,1]$; $[3,1]$; $[1,3]$; and $[3,3]$; with the closed section of SqWTE and other remaining combinations, while there one or both exponent has even value gives open section SqWTE and they all are called even superquadrics here in this paper.

CHAPTER 5: RESULTS AND DISCUSSION

5.1 Advancement of the presented Work

The presented work extends the family of superquadrics and achieved new shapes and one new parameter called the third exponent and a valid mathematical and geometrical modeling of equations inclusive of this third exponent are presented under this thesis. On alteration for airtight geometrical modeling are suggested for the previous mathematical equation of superquadrics when using Mathmod 9.1. A Gridded Superquadrics also introduced with distinct shapes and sizes of models corresponds to the distinct size of grids and quantity of polylines.

5.2 Applications of the findings

Superquadrics have a wide range such as in the modeling of robots [12], airplanes, automobiles, and animals [11]. It is also useful in the designing and/or modeling of various games, realistic operations, and complex objects. Since this may apply to both static and dynamic parts. Therefore it can be used in various filed of design engineering where mathematical modeling is possible.

5.2.1 Application of PMSq

The nonformulaic models which have been discussed in section 3, whether rational or irrational, are looking useful in the world of primitives and also for the representation of matching objects in computer graphics and physical modeling too. Such as in the modeling of complex machinery, robotics [7] video games, Animated video, Animated presentation, simulation of operations, volumetric representation [8], shape recovery, various segmentation [4] and at other many more places in the field of geometrical designing and modeling.

5.2.2 Application of TESq

SqWTE are discussed in section 2, and also represented a classification of these models of SqWTE based on even and odd values combination of the second and the third exponent, called as even and odd superquadrics (ESq and OSq), that gives an extension to the superquadrics family and may apply in designing, modeling, and reconstruction of complex objects as well as previous superquadrics models, discussed by A.H. Barr [1] and D. Paschalidou [11] for the field of computer graphics, robotics, aerospace and mechanical industry too.

5.3 **Limitation**s

We have seen the limited geometrical models for exponent value for the combination up of exponent value greater than 4.0, If we taking the numerical value above than 4.0 for all exponents then we don't get solid models or surfaces.

CHAPTER 6: CONCLUSIONS AND CLOSING

By the study on NURBS & Superquadrics to improve the quality aspects of design and model of complex objects and the review of earlier published research papers & books in such fields, Some important concluded points are listed as below, Mathematical modeling is an advanced method to model a complex object easily. Which allows making any alteration within a moment by changing the specified parameter(s).

It is less time & cost consumable modeling method compare to others. Superquadrics have a great scope in such a modeling method. As it's a set of a lot of shapes. And can obtain by a single equation by changing the parameter(s)' value. In Superquadrics size & shape of models varies with numbers (quantity) and size of grids. Superquadrics equations may not valid for some combinations of both exponents & grids. Resetting the limits and variables gives the different shapes and sizes of models. Surface quality and morphology varies with the number, values & given limits of such variables. Superquadrics are useful in designing and modeling of complex structure. Superquadrics generalize the basic quadric surfaces and solids. Angle-preserving transformations, bending and twisting make a new form of the object. New primitives and operators have potential design applications wherever flexible operations are needed, or where volume, surface area, or arc length must be conserved they provide a powerful extension to the classical design shapes [10]. The primitives of superquadrics have a wide range of applications in representing complex objects and have much fewer parameters than meshes [12]. The selection of a surface model to represent and visualize the body geometry is of crucial importance for contact analysis. The most important aspects to take into consideration are the geometric representatively of the surface and the analyticity of the surface functions. Quadric and superquadrics surfaces are geometric descriptions that are used to model a large variety of 3-D shapes [14]. In the account of advantages, easily can say that It provides simple expressive geometry. It makes easy for the reconstruction of a complex object.

Superquadrics enable the improvement in both the process and quality aspects of various designs and modeling of complex 3D objects in both real and computer graphics world. It has an amazing world of primitives that can be made easily by mathematical modeling and also can be modified. So, it provides a simple way to design and modeling of complex objects. Some Important findings of this study are summarized as follows,

1. Mathematical modeling is an advanced method to model a complex object easily.
2. This allows many alterations within a moment by changing the specified parameter(s).
3. It is less time & cost consumable modeling method due to using of mathematical formulation.

4. Superquadrics have a great scope in the design and modeling of complex objects.
5. Superquadrics equations may not valid for combinations of higher values of both exponents.
6. Resetting the limits and variables gives various shapes and sizes of models.

As above, we saw the availability of large no. of various extended models under the superquadrics equations using third exponent and grid size, as given the examples in section 2 for Sq-WTE, for few compositions of exponent value; and in section 3 for GSq, for two compositions of exponent value. It is concluded that a valid extension of two types, in the family of superquadrics has obtained, which gives a lot of shapes and sizes for a single mathematical equation, which needs to justify and mapped will do in the future. Yet, according to the above, some important notes are concluded as below,

1. It is found that the third exponent gives another new look to the superquadrics family and therefore it is useful in various applications like previous superquadrics models.
2. In Mathematical Modeling of Superquadrics, Grid Density, and Poly No. giving various models separately and in exponent combination too.
3. Even, some models of them are irrational and all other except a mature value range of Grid Density and Poly no. are yet nonformulaic.
4. It has great scope and a large work for the researcher in this field may summarise as to check the models for various exponential combinations with third exponent, grid size, limits, and angular variation.
5. After remapped them into a meaning full, systemic, and scientific way.
6. By which, they can be used at different places and to ensure the availability of such all valid models easily on a click.

Thus the introduced concepts, Superquadrics with Third Exponent (Sq-WTE) and Gridded Superquadrics (GSq) are found a valid extension of the family of Superquadrics.

Further, we plan to check the models for cylindrical coordinates, with consideration of the third angle in addition to the third exponent. And, also we are looking to check the availability of other remaining gridded superquadrics (GSq). That belongs to the family of superhyperboloids and supertoroids, and, also planning to summarize all these models scientifically.

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Curriculum Vitae

Hem Raj Yadav

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➢ Career Objective:-

As I have the interest to work for Invention, Research & Development in Science, Social Science, Economics, Engineering & Technology from my childhood. Therefore, I aspire to become a great performer to serve those kinds of institution and society where the people evolve together, regardless of whether they connected directly or indirectly with the organization and which ensures the development and welfare of the whole world in the field of intellectual, technical, physical, social, economical, spiritual, psychological and overall. Thus for my whole life, I believe that it is the right and best platform, where I can ensure a respectfully and great life and can contribute my best for society too. Hence, my best endeavour may assume for such a role with higher probability.

➢ Educational Qualification:-

Qualifying Examination	Pass Out Year	College	Board/Univ.	Subject / Branch	Result (In %)
High School	2003	Shree Shiv Pratap Intermediate College Amethi	Uttar Pradesh Board of High School & Intermediate Allahabad (U.P. Board)	Hindi, English, Science, Social Science, Sanskrit & Mathematics	57.5%
Intermediate	2005	Shree Ranveer Intermediate College Amethi	Uttar Pradesh Board of High School & Intermediate Allahabad (U.P. Board)	Hindi, English, Physics, Chemistry & Mathematics	50.4%
B.Sc.	2011	Shree Ranveer Rananjay Post Graduate College Amethi (RRPG)	Dr. Ram Manohar Lohia Avadh University Faizabad	Physics, Chemistry, Mathematics	45.9%
B. Tech. (AE)	2015	Babu Banarasi Das National Institute of Technology & Management Lucknow (BBDNITM)	Uttar Pradesh Technical University Lucknow (U.P.T.U. Lucknow)	Aeronautical Engineering (AE)	73%

M. Tech. (ME)	2020 *	School of Engineering	Babu Banarasi Das University (BBDU) Lucknow	Design Engineering	80%**
*As estimated, **It is only 1st Year Result as the course is pursuing.					

➢ Professional Training & Short Term Course:-

- **One-Year** (July 2007 to Oct 2008) diploma Titled ***"Computer Accountant"*** from ***"Academy Of Computer Accountant (ACA)"*** Tilak Nagar Kanpur Branch of ACA Jaipur.

- **One-Year** Certificate Course Titled ***"Commercial Manager"*** From ***"Academy of Computerized & Financial Accounting (ACFA)"*** Kanpur.

- **Three-Month** (From Sep 2013 to Dec 2013) Training in ***"MATLAB Programming & Flight Mechanics Simulation"*** organized by *"**Orane Lab IIT Kanpur"*** at BBD Lucknow.

- **Three-Month** Course titled ***"AUTOCAD SOLID WORKS"*** From ***"RVM CAD Lucknow"***.

- **One-Month** (From 17th Jun 2014 to 16th Jul 2014) ***"Technical Training"*** with ***"Indira Gandhi Rashtriya Uran Academy (IGRUA)"*** Fursatganj Airfield Amethi in the maintenance of Air Craft Engine, Cockpit Instruments and Air Frame.

- **One-Month** (From 15th Jun 2013 to 27th July 2013) ***"Technical Training"*** with ***"Fasteners India"*** Rae Bareli in Fabrication, Machine Design, Manufacturing Process and Operations etc.

➢ Projects:-

- ✓ ***B. Tech. Thesis*** executed along with the title ***"Introduction to Air Vehicles, Space Vehicles and Rockets & Missiles"*** under the guidance of ***"Prof. C.C. Gupta"*** HOD, Department of Aeronautical engineering, BBDNITM, Lucknow.

- ✓ A ***Project*** with the title ***"CFD Design Analysis of Combustion Chamber and Gas Turbine Engine"*** under guidance of ***"Associate Prof. Parmendra Singh"***, Department of Aeronautical Engineering, BBDNITM, Lucknow executed by me.

➢ Skills:-

- ***Academic Subject:*** Fluid Mechanics, Flight Mechanics, Aerodynamics, Machine Design, Propulsion, Aircraft Structure, Strength of Material, Mathematics and Thermodynamics.

- ***Specialization in:*** Design Engineering

- ***Computer Software:*** Auto CAD, Catia, Solid Works, Ansys, MATLAB, NI Lab View, MS Office, Tally, Busy etc.

- ❖ ***Industrial Capability:*** Capable to handle maintenance of Air Frame, Structure, Piston & Jet Engines, Cockpit Instruments and other auxiliary systems installed on Aerospace Vehicles.

- ❖ ***Project Capability:*** Capable to handling Engineering Project / work related to Research, Development, Production and Maintenance as a Project Engineer in the field of Aeronautical / Mechanical Engineering.

- ❖ ***Proficiency:*** Proficient in Aeronautical Engineering like as Aerodynamics of Aircrafts, Aircraft Structures, Propulsion (Gas Turbine Engines and Auxiliary Power Units, Afterburner etc).

➢ Work Experience:-

✓ **Two-Year** (2005-2007), as ***"Teaching experience"*** on the level of tuition & coaching.

- **Subject**: Physics, Chemistry, Mathematics, Computer Software (Basic, MS Office, Tally & Busy etc.).

✓ **Three-Year** (2007-10), as ***"Assistant Accounts Manager"*** with ***"Garg Gas Service"*** Kanpur.

- **Responsibilities:** Maintain Accounts book with Tally and manual Registers, Statutory Returns Filing, Finalization with CA, Banking operation, Payment and Recovery, Sales and Customer handling, other HR & Admin Work, Reporting to Proprietor.

✓ **Three-Year** (2010-2013), as ***"Asst. Production Manager cum Accountant"*** *w*ith ***"Fasteners India"*** Rae Bareli.

- **Responsibilities:** Production Schedule & Planning, Purchases Estimation, Budget Preparation, Time Scale with Clients, Production Management, Quality and Control, Stock Analysis, Human Power Management, Inventory Management, Sales Analysis, Maintain Accounts Book with Tally ERP 9.0, Statutory Returns Filing, Finalization with CA, Banking operation, Payment and Recovery, ITR Filing, Reporting to Proprietor and Production Manager.

✓ **Three-Month** (Aug 2015 to Oct 2015), as ***"Customer Service Associate (Voice)"*** Under EFL Process with ***"SERCO BPO PVT. LTD."*** Goregaon (E) Mumbai.

- **Responsibilities:** Answer Incoming Calls, Addressing customer complaints, Customer satisfaction with good conduct & delivery.

✓ **One-Year** (2015-16), as ***"Accountant cum Manager"*** with ***"JP Enterprises and RK Air conditionings"*** Kandivali East Mumbai.

- **Responsibilities:** Service & Installation Schedule & Planning, Purchase Estimation, Budget Preparation, Sales Analysis, Stock Analysis, Human Resource Management, Maintain Accounts Book with Tally ERP 9.0, Statutory Returns Filing, Finalization with CA, Banking operation, Payment and Recovery Reporting to Proprietor.

✓ **Two-Year** (From 15th Feb 2016 to 28th May 2018), as ***"Sr. Executive-HR, Administration & Indirect Taxation"*** with ***"GKB Hi – Tech Lenses Pvt. Ltd."*** Mumbai**.**

- **Responsibilities:** HR Management, Admin operations, Recruitment, Statutory Licensing and returns filing, Legal Cases, Indirect Taxation of Maharashtra and Gujarat State, Reporting to Respective Managers – Head office Goa.

✓ **Two-Month** (From 27th Aug 2018 to 10th Oct 2018), as ***"'Accounts Executive"*** with ***"Achievers Credit Co-operative Society Ltd."*** Lucknow**.**

- **Responsibilities:** Accounts Entry, Banking Work, Payment and Receivables, Reports Preparation and Cash Handling and so on, Reporting to Accounts Head.

➢ **Expected stipend:** More than ₹ 3.5 Lac Per Annum.

➢ Registered and Published Work:-

❖ A ***Book*** under the Title **"Introduction to Aerospace Vehicles"** published with ***"Notion Press"*** in 2019, URL: https://notionpress.com/read/introduction-to-aerospace-vehicles

❖ A ***Review Paper*** has been published under the title ***"Role of Response Surface modeling in Manufacturing Operation – An Overview"*** in National Conference on ***"Futuristics in Mechanical Engineering (FME-2019)"*** Sponsored by ***"TEQIP-III"***, Organized by ***"Department of Mechanical Engineering, MMMUT"*** Gorakhpur and published by ***"ELK Asia Pacific Journals"*** New Delhi- 110034. Pages 130-143 (373). URL: https://www.elkjournals.com/microadmin/UploadFolder/619ROLE-OF-RESPONSE-paper-19.pdf

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❖ **Registered** more than hundred ***Hindi Lyrics*** with ***"Screen Writer Association"*** Mumbai from 2016 to onwards.

➢ Personal Profile:-

Name	:	Hem Raj Yadav
Date of Birth	:	10th Dec 1987 A.D.
Gender	:	Male
Marital Status	:	Married (w.e.f. 26th Jun 2012)
Father	:	Shree Siya Ram Yadav (Lineman in KESCO)
Mother	:	Smt. Budhna Devi
Spouse	:	Mrs. Krishna Yadav
Daughter	:	Bhoomi Manav
Language	:	Hindi & English

➢ Quality, Strength & Nature:-

❖ Positive thinking & attitude towards life and working

- ❖ Working with responsibility and honesty
- ❖ Having Good Inter-Personal and communication skills
- ❖ Self-dependency in working
- ❖ Ability to lead a team of person with good conduct

➢ **Hobby:** Writing Lyrics and Books

- ✓ Member of Screen Writer Association Mumbai w.e.f. 11th Apr 2016 to onwards.
- ✓ To enjoy a part of my writing please visit https://www.yourquote.in/hemrajyadav95

✓ **Declaration:-** I hereby declare that all the information furnished above are true and genuine to the best of my knowledge.

Place: Lucknow
Date: 27th May 2020

(Hem Raj Yadav)